Public Works (189) Department.

G.O., No. 2938 W

DATED 6th Nov 89

[Despatch Abstract.]

Indents for Rolled Steel Joists.

Circular from the Govt of India forwarding certain suggestions for the preparation of Indents by Officers of the P.W.D

T/B 280/14

Despatches to note 24/11

[P. W. 26. (85)—26-4-97.]

BACK PAPERS.

G.O.	No.	Department.	Date.	G.O.	No.	Department.	Date.

FURTHER PAPERS.

G.O.	No.	Department.	Date.	G.O.	No.	Department.	Date.

GENERAL.
ACCOUNT.

Government {1899} of Madras.

PUBLIC WORKS.

Recd. } 1899. *Enclosures*

Regd. *Spare copies*

G.O., No. 2938 W., 6th November 1899.

NOVEMBER.

Indents for Rolled Steel Joists

Circular from the Government of India forwarding certain suggestions for the preparation in India of —, communicated to officers of the Public Works Department.

GOVERNMENT OF MADRAS.

PUBLIC WORKS DEPARTMENT.

General.
Account.

Read—the following paper:—

From the Government of India, Public Works Department (Railway Stores), dated Simla, 20th September 1899, Circular No. 4 Ry.

The Secretary to the Government of Madras, Public Works Department.
The Joint Secretary to the Government of Madras, Public Works Department, Railway Branch.
The Secretaries to the Government of Bombay, Public Works Department, General and Railway Branches.
The Secretaries to the Governments of Bengal, the North-Western Provinces and Oudh, and the Punjab, Public Works Department.
The Secretaries to the Government of Burma, Public Works Department, General and Railway Branches.
The Chief Commissioners of the Central Provinces, Assam, and Coorg.
The Resident at Hyderabad.
The Resident in Mysore.
The Agents to the Governor-General for Central India and Rajputana, and Baluchistan.
The Accountant-General, Public Works Department.
The Director-General of Military Works.
The Consulting Engineers to the Government of India for Railways, Calcutta, Lucknow, and Assam.
The Director-General of Telegraphs.
The Managers, North-Western, Oudh and Rohilkhund, East Coast and Eastern Bengal State Railways.
The Engineer-in-Chief, Hardwar-Dehra Railway.
The Engineer-in-Chief, Indus Bridge Works and Shadipali-Balotra Railway.
The Engineer-in-Chief, Gódávari Bridge Works.
The Engineer-in-Chief, Ghaziabad-Moradabad Railway.
The Port Storekeeper, State Railway Department, Calcutta.

The under-mentioned paper is forwarded to the Local Governments and Administrations and to the officers noted on the margin for information and guidance, in supersession of Government of India Circular No. 2 Ry. of 25th January 1895.

2. These revised instructions are applicable to all branches of the Public Works Department.

(By order.)

(Signed) C. J. COLE,
Under Secretary.

To the Secretary to the Government of Madras,
Public Works Department.

Enclosure.

Suggestions for the preparation in India of Indents for Rolled Steel Joists.

(Revised, December 1898.)

1. Difficulty is sometimes experienced in procuring rolled joists of the exact sections demanded. It is therefore recommended that a detailed statement, giving limits of variation and other particulars for each item, should be furnished on the form shown on page 3.

2. Joists are tested with central loads and must show no permanent set after test. The loads are arranged so as to produce a limiting stress of 7½ tons per square inch. Samples cut from the joists are required to bear an ultimate tensile stress of 27 to 32 tons per square inch, with an elongation of 20 per cent. in a length of 8 inches.

3. Steel joists are now invariably supplied, because "open hearth" mild steel has practically supplanted wrought-iron for structural work.

4. Taking the modulus of elasticity for steel at 12,500 * tons per square inch, and that for iron at 11,125 † tons per square inch, the deflection of a steel joist with a limiting stress of 5·62 tons per square inch is the same as that of an iron joist of the same section, and under the same conditions, with a limiting stress of 5 tons. With higher limits of stress, as 7½ tons, the deflection is, of course, increased.

5. Table I, annexed, gives particulars of various sections, together with constants for each, from which loads and deflections can be easily ascertained. The sections enumerated are all stated in published lists to be "stock" sections, and should therefore be readily obtainable, but special rolling and consequent delay are generally found necessary, probably because contractors will not cut stock lengths to waste.

6. In estimates of weight, 2 per cent. should be added to list weights of joists to allow for variations in rolling.

7. Tables II and III which follow the Appendix are intended to reduce clerical labour in cases where built-up plate girders have to be designed. Table IV gives particulars of bending moments, etc., of beams under various conditions.

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* Professor T. A. Hearson, M.I.C.E. | † Molesworth's Pocket Book.

Detail of Rolled Steel Joists demanded on Indent _____

Item of Indent.	Section required. Depth.	Width.	Weight per foot run.	Maximum and minimum limits of dimensions of section which may be allowed in case the exact section cannot be procured. Depth. Max.	Depth. Min	Width. Max.	Width. Min.	Length.	Clear span.	Number required. In dead lengths, *i.e.*, both ends machined.	With one end machined.	With neither end machined.	Dead load, including weight of joist.	* Arrangement of load.	† Condition of joist.	Limiting stress per square inch.	Remarks. State whether built-up plate girders may be substituted for joists, and, if so, whether they must be of uniform *section* throughout or may be of uniform *strength and depth*.
	IN.	IN.	LB.	IN.	IN.	IN.	IN.	FT. IN.	FT. IN.				TONS.			TONS.	

* *i.e.*, Distributed.
Central.
etc.

† *i.e.*, Fixed both ends.
Supported, ditto.
etc.

No. 2938 W., 6th November 1899.

TABLE I.

Rolled Steel Joists.

Particulars of Sections, and Constants for Load and Deflection, with a limiting Stress of 7·5 tons per square inch.

Case I.

Condition of joist ... Supported at both ends.
Arrangement of load ... Dead, uniformly distributed, weight of joist included.

Section.	Weight per lineal foot.	Thickness of web.	Moment of inertia.	Constants. For load.	Constants. For deflection.
INCHES.	LB.	INCH.	QUARTIC INCHES.		
20 × 7½	89	·60	1,646·6	822·8	·09
19¾ × 7¼	96	·91	1,715·48	868·60	·091
18 × 7	75	·55	1,150·53	639·2	·10
18 × 6⅞	80	·625	1,163·05	656·21	·101
17½ × 6	62	·56	722·36	451·5	·1125
16 × 5	50	·51	566·01	347·50	·1125
15¾ × 6½	62	9/16	704·58	447·83	·114
15 × 6	59	·54	617·05	411·37	·12
15 × 5½	56·5	9/16	572·64	381·76	·12
15 × 5	42	·422	414·51	276·34	·12
14 × 6	57	·51	5:0·65	371·9	·1286
14 × 6	46	·435	422·57	301·88	·1286
13¾ × 5½	46	7/16	419·63	299·73	·13
13 × 5	41·5	·51	300·65	231·27	·138
12 × 6	64	·61	869·91	308·27	·15
12 × 6	44	·41	314·4	262·00	·15
12 × 6	42	·42	296·02	246·68	·15
12 × 5	39	·44	254·48	212·07	·15
12 × 5	32	·35	220·95	184·13	·15
11⅞ × 5	37	·42	242·46	205·25	·153
10 × 6	45	·489	215·97	159·97	·18
10 × 5	35	·48	159·97	159·97	·18
10 × 5	29	·35	141·67	141·67	·18
10 × 4½	30	·387	141·42	141·42	·18
9⅞ × 6	41	·42	200·23	202·76	·182
9½ × 5	27	·36	125·90	127·50	·182
9 1/16 × 4½	36	·516	159·22	158·19	·184
9 1/16 × 4	22	·33	80·64	88·87	·198
9 × 7	58	·777	216·81	240·90	·20
9 × 3½	20	·30	75·02	83·36	·20
8 × 6	35	·44	111·59	139·49	·225
8 × 6	32	7/16	101·25	126·56	·225
8 × 5	30	·40	93·86	117·33	·225
8 × 4	25	·41	73·24	91·55	·225
8 × 4	19	·329	56·48	70·60	·225
7⅞ × 5	27	·375	82·48	104·74	·2286
7½ × 4	19	7/16	56·16	71·31	·2286
7 × 3½	18	·313	41·99	59·99	·257
7 × 3½	16	·25	35·98	51·4	·257
7 × 3½	16	·25	38·80	55·43	·257
6½ × 3½	18	·339	33·09	52·94	·288
6½ × 2	9·5	·25	15·81	25·29	·288
6 × 5½	24	7/16	40·47	66·40	·294
6 × 5	25	·423	43·46	72·43	·30
6 × 4½	20	·434	33·35	55·58	·30
6 × 3½	13	·25	26·79	44·65	·30
6 × 3	16	·39	25·14	41·90	·30
6 × 3	13	·322	20·77	34·62	·30
6 × 2	12	·381	16·58	27·64	·30
5½ × 3	10 5	·329	12·9	23·15	·327
5½ × 2	9	·368	8·77	16·07	·327
5 1/16 × 4½	17·5	5/16	23·14	46·15	·351
5 × 5	24	·371	29·55	59·10	·36
5 × 5	22	·342	26·94	53·88	·36
5 × 4 7/16	19	·44	22·09	44·18	·36
5 × 3	15	·40	16·09	32·18	·36
5 × 3	11	·23	13·65	27·30	·38
5 × 3	10	·25	11·93	23·86	·36
5 × 2	9·5	·25	10·72	21·44	·36
4⅞ × 3	10	·40	8 43	16·86	·379
4½ × 1 15/16	7·5	3/8	6·50	13·68	·379
4½ × 3	14	·21	7·67	15·36	·384
4⅜ × 3	10	·40	8·42	16·84	·39
4 × 3	12	·23	9·03	20·41	·436
4 × 3	9·5	·299	7·538	22·58	·45
4 × 2	6·5	·225	4·23	18·85	·45
4 × 1 13/16	8	3/16	5·16	10·6	·45
4 × 1 14/16	5	·331	3·628	12·9	·45
3½ × 3	10·5	·18	5·87	9·07	·45
3½ × 1 15/16	6	·35	2·91	16·77	·514
3½ × 1 15/16	4	·226	1·80	8·31	·614
3½ × 3 7/16	8	·17	3·67	5·76	·576
3 × 1½	10	·25	4·21	11·98	·588
3 × 1½	4·5	·29	1·90	14·03	·60
3 × 1½	4	·156	1·47	6·33	·60
		·218		4·90	·60

To use Table I.

(i) To find the load and corresponding deflection for a given section of joist with a given span :—

(*a*) Divide the constant for load by the distance *in feet* from centre to centre of bearings ;* the quotient is the total distributed load *in tons*, including the weight of the joist. †

(*b*) Multiply the constant for deflection by the square of the distance *in feet* from centre to centre of bearings,* and divide the result by 100 ; the quotient is the deflection in inches.

(ii) To select a joist for a given external load (*i.e.*, load exclusive of weight of joist) and given span :—

Multiply the load *in tons* by the distance *in feet* from centre to centre of bearings,* and take a section against which the constant for load is slightly higher than the result. The weight of the joist† should then be added to the external load, and the section tried as in (i) for load and deflection. The depth of a joist should not, as a rule, be less than from $\frac{1}{15}$ to $\frac{1}{25}$ span, and in the case of joists without lateral support, *e.g.*, the gantry girders for a traveller, the width should not be less than $\frac{1}{30}$ span.

Cases II to XIII.

(iii) To ascertain loads, etc, in these cases, the constants in Table I must be multiplied by the factors given below for each particular case before proceeding as in (i) or (ii) above. In Cases IX to XIII the symbol L=distance in *feet* from centre to centre of bearings :—

Case.	Condition of joist.	Arrangement of load (dead in all cases).	Factors to be applied to	
			Load constant.	Deflection constant.
II	Fixed at both ends	Uniformly distributed	1·5	·3
III	Fixed at one end, supported at the other	Ditto	1	·415
IV	Fixed at one end, free at the other ..	Ditto	·25	2·4
V	Supported at both ends	Concentrated at centre	·5	·8
VI	Fixed at both ends	Ditto	1	·4
VII	Fixed at one end, supported at the other	Ditto	$\frac{3}{8}$	·475
VIII	Fixed at one end, free at the other ..	Concentrated at free end ..	·125	3·2
IX	Supported at both ends	Half $\left(\frac{W}{2}\right)$ concentrated at each of two points equidistant from the centre ; each point A feet from the nearer bearing, and B feet from the further bearing. (A + B = L.)	$\frac{L}{4A}$	$\frac{6L^2-8A^2}{5L^2}$
X	Fixed at both ends		$\frac{L^2}{4AB}$	$\frac{6LA-8A^2}{5LB}$
XI	Fixed at one end, supported at the other		$\frac{L^2}{6AB}$	‡
XII	Supported at both ends	Concentrated at one point, A feet from the nearer, and B feet from the further bearing. (A + B = L.)	$\frac{L^2}{8AB}$	$\frac{16AB}{5L^2}$
XIII	Fixed at both ends		$\frac{L^3}{8AB^2}$	$\frac{16A^2B}{5L^3}$

* In Cases IV and VIII substitute "centre of bearing of fixed end to free end" for "centre to centre of bearings."

† In Cases V and VIII, VI and VII, IX, X and XI, XII, XIII: Total load = external load + $\left\{\frac{\frac{1}{2}}{\frac{3}{8}}, \frac{L}{4A}, \frac{L^2}{6AB}, \frac{L^2}{8AB}, \frac{L^3}{12AB^2}\right\}$ × weight of joist.

In these cases the actual weight of the joist is not, for purposes of calculation, added to the external load to make up the total load, because the calculations would be made for concentrated loads, whereas the weight of the joist would be uniformly distributed. The calculated deflections will, in these cases, not be quite correct ; and, in Cases XII and XIII, they will represent the deflection at the point where the load is applied, and not the maximum deflection. Nearer approximations may, however, be obtained by using the deflection formulæ given in Table IV, as explained in Note (ii) at the foot of pages 14 and 15.

‡ This factor is omitted as it cannot be stated shortly. *See* Case XI in Table IV.

Notes to Table I.

(*a*) The constants given in the table can readily be adapted to suit other limits of stress than that which has been used (7·5 tons per square inch), *e.g.*, if it be desired to limit the stress in a joist to 6 tons per square inch, the constants must be multiplied by $\frac{6}{7\cdot5}$ or 0·8 before the load and deflection are determined.

(*b*) If the deflection calculated from the table be greater than can be allowed, and if the circumstances do not permit the use of a deeper section or of a heavier section of the same depth, the load calculated from the table must be reduced in the proportion of the allowable to the calculated deflection and the number of joists increased, *e g.*, if the allowable deflection be 0·5 inch and the calculated deflection 0·6 inch, the load should be reduced to $\frac{5}{6}$ths of the calculated load ; the stress would, of course, be reduced in the same proportion.

(*c*) If the deflection be limited to $\frac{1}{400}$th of an inch per foot run of the joist, the minimum depth (with the limit of stress used in the table) should be as follows for Case I, *viz.*, ·72 inch for each foot of distance between centres of bearings ; *e.g.*, if the distance from centre to centre of bearings were 10 feet and the deflection were limited to $\frac{1}{400}$ or $\frac{1}{4}$ inch, the minimum depth of joist, Case I, should be 7·2 inches.

APPENDIX.

The constants in Table I have been calculated from the following data :—

Symbols.

W = Total distributed load in *tons*, including the weight of the joist.
L = Distance in *feet* from centre to centre of bearings.
S = Limiting stress in *tons* per square inch ($7\frac{1}{2}$ for mild steel).
d = Depth of joist in *inches*.
V = Deflection in *inches*.
E = Modulus of elasticity in *tons* per square inch (12,500 for mild steel).
* I = Moment of inertia of section of joist (from dimensions in inches).

Formulæ.

For *distributed* loads, ends of joists *supported* (Case I),—

$$\tfrac{1}{8}\,WL \times 12 = \frac{SI \times 2}{d}.$$

or $WL = \dfrac{4SI}{3d}$ (1) which is constant for each section for any fixed value of S.

$$V = \frac{5}{48} \times \frac{SL^2 \times 12^2 \times 2}{Ed} \text{(2)}.$$

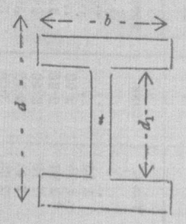
From (2) $\left(\dfrac{V}{L^2}\right) = \dfrac{30S}{dE}$ (3) which is constant for each *depth* of joist for any fixed values of S and E.

For steel S = 7·5. E = 12500.

From (1) $WL = \dfrac{10I}{d}$. (Tabular constant for load.)

From (3) $\dfrac{\dagger\, 100V}{L^2} = \dfrac{1{\cdot}8}{d}$. (Tabular constant for deflection.)

* When the thickness of the web is known as well as the depth, breadth, and weight per foot lineal, the moment of inertia for the section may be very approximately determined as follows :—
Let A = area of the section in square inches.
,, w = weight of the section in lb. per foot lineal.
Then for steel $A = \dfrac{w}{3{\cdot}4}$, because 1 foot lineal of steel 1″ × 1″ weighs 3·4 lb.
Assume the flanges to be of equal thickness throughout, *i.e.*, that the section is rectangular, as shown in figure,—



Then $I = \dfrac{1}{12}\left\{bd^3 - (b-t)\,d_1^{\,3}\right\}$, in which d_1 is the only dimension that is not given.
From the figure, $A = bd - (b-t)\,d_1$
whence $d_1 = \dfrac{bd - A}{b-t}$
and as $A = \dfrac{w}{3{\cdot}4}$; d_1 can be found and the value of I determined.

In manufacturers' lists of joists, it is generally stated that joists can be rolled about 10 per cent. heavier than list weights ; the extra weight in such cases is obtained by slightly opening the rolls and thus thickening the web, and the consequent addition to the tabular constant for load is $\dfrac{xd}{4{\cdot}08}$ for a steel joist, x being the extra weight in lb. per foot lineal of the joist and d its depth in inches. The calculation is as follows :—

* Lest C = increase in area (in square inches) due to increase of x lb., per foot lineal, in weight. Then $\dfrac{C}{d}$ = increase in thickness of web.

For the rectangle $d \times \dfrac{C}{d}$ $\begin{cases} I = \dfrac{1}{12}\,d^3 \times \dfrac{C}{d} = \dfrac{Cd^2}{12}. \\ \text{and from equation (1) above, } WL = \dfrac{4SI}{3d} = \dfrac{SCd}{9}. \end{cases}$

For steel, $S = 7{\cdot}5$, $C = \dfrac{x}{3{\cdot}4}$, and $WL = \dfrac{xd}{4{\cdot}08}$ or approximately $\dfrac{xd}{4}$.

† The object of multiplying by 100 is to save long decimals in the tabular constants for deflection.

TABLE II.

SAFE Resistances of Mild Steel Rivets (for Girders) to Shearing and Crushing.

Limiting Stresses. { Shearing, $5\frac{1}{4}$ tons per square inch of section. / Crushing, 8 tons per square inch of bearing area.

If d = diameter of rivet, and t = thickness of plate ; then Bearing Area = dt.

Diameter of rivet.	Resistance to Shearing. Single shear.	Resistance to Shearing. Double shear.	Resistance to Crushing. Thickness of plate through which rivet passes—in inches. $\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	1	Weight per 100 of rivet heads—snap or cup.
INCH.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	TONS.	LB.
$\frac{3}{8}$	·61	1·21	·75	·94	1·13	1·31	1·50	1·69	1·88	2·06	2·25	2·44	2·63	1·47
$\frac{7}{16}$	·83	1·65	·88	1·10	1·31	1·53	1·75	1·97	2·19	2·41	2·63	2·84	3·06	2·34
$\frac{9}{16}$	1·08	2·16	1·00	1·25	1·50	1·75	2·00	2·25	2·50	2·75	3·00	3·25	3·50	3·48
$\frac{11}{16}$	1·37	2·73	1·13	1·41	1·69	1·97	2·25	2·53	2·81	3·09	3·38	3·66	3·94	4·96
$\frac{3}{4}$	1·69	3·37	1·25	1·56	1·88	2·19	2·50	2·81	3·13	3·44	3·75	4·06	4·38	6·79
$\frac{13}{16}$	2·04	4·08	1·38	1·72	2·06	2·41	2·75	3·09	3·44	3·78	4·13	4·47	4·81	9·04
$\frac{7}{8}$	2·43	4·86	1·50	1·88	2·25	2·63	3·00	3·38	3·75	4·13	4·50	4·88	5·25	11·74
$\frac{15}{16}$	2·85	5·70	1·63	2·03	2·44	2·84	3·25	3·66	4·06	4·47	4·88	5·28	5·69	14·93
1	3·31	6·61	1·75	2·19	2·63	3·06	3·50	3·94	4·38	4·81	5·25	5·69	6·13	18·63
$1\frac{1}{16}$	3·79	7·59	1·88	2·34	2·81	3·28	3·75	4·22	4·69	5·16	5·63	6·09	6·56	22·92
$1\frac{1}{8}$	4·32	8·64	2·00	2·50	3·00	3·50	4·00	4·50	5·00	5·50	6·00	6·50	7·00	27·82
$1\frac{3}{16}$	4·88	9·75	2·13	2·66	3·19	3·72	4·25	4·78	5·31	5·84	6·38	6·91	7·44	33·87
$1\frac{1}{4}$	5·47	10·93	2·25	2·81	3·38	3·94	4·50	5·06	5·63	6·19	6·75	7·31	7·88	39·61

Example of the use of this table :—
Given, { Thickness of web of girder—$\frac{7}{16}$-inch. / Pitch of rivets—4 inches ; rivets in double shear. / Shearing stress—9 tons per foot of depth, or per foot run.
Required.—The diameter of the rivets necessary to resist crushing and shearing.
As there are three rivets per foot, the resistance per rivet must not be less than $\frac{9}{3}$ or 3 tons : in the column for $\frac{7}{16}$-inch plates, 3·06 tons, the resistance to crushing next above 3 tons is found in the line for $\frac{3}{4}$-inch rivets ; the diameter required is therefore $\frac{3}{4}$ inch. As the rivets are in double shear, their resistance to shearing is more than sufficient.

TABLE III.

Safe Resistances of Web Plates of Mild Steel Girders to Buckling. Calculated from the Formula of Professor T. C. Fidler, M.I.C.E., for columns of Mild Steel with Fixed Ends.

Clear unsupported distance between flange angles or between vertical stiffeners.	Thickness of web plate—in inches.									Explanations.
	$\frac{1}{4}$	$\frac{6}{16}$	$\frac{8}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	
INCHES.	Safe shearing stress per foot in height of web—in tons.									
12	4·17	7·50	11·57	16·33	21·12	26·19	31·13	36·03	40·50	Examples of the use of this table :—
13	3·60	6·56	10·44	14·92	19·68	24·71	29·63	34·49	39·15	
14	3·15	5·81	9·36	13·49	18·36	22·95	28·05	33·00	37·80	(i) *Given.*—Depth of girder, 36 inches ; vertical distance between flange angles, 28 inches ; total shearing stress, 18 tons.
15	2·82	5·21	8·51	12·39	16·92	21·53	26·40	31·60	36·45	
16	2·52	4·65	7·65	11·29	15·42	20·05	24·90	30·03	34·92	*Required.*—The thickness of the web to resist buckling when no stiffeners are used.
17	2·22	4·13	6·89	10·34	14·28	18·97	23·55	28·29	33·39	
18	1·92	3·75	6·26	9·45	13·26	17·35	22·13	26·81	31·68	
19	1·77	3·49	5·72	8·61	12·18	16·34	21·00	25·49	30·15	The shearing stress per foot in height of the web $= \frac{1}{2}^{8} = 6$ tons ; in the table on the line for a clear distance of 28 inches between flange angles, the nearest stress to 6 tons is 6·30 tons, which occurs in the column for $\frac{7}{16}$-inch plates. The thickness required is therefore $\frac{7}{16}$ inch.
20	1·65	3·15	5·18	7·93	11·34	15·12	19·28	24·17	28·71	
21	..	2·85	4·73	7·30	10·32	14·04	18·15	22·44	27·54	
22	..	2·55	4·41	6·77	9·60	13·10	16·95	21·20	26·28	
23	..	2·29	4·14	6·20	9·00	12·35	15·98	20·05	24·48	
24	..	2·19	3·78	5·72	8·34	11·48	15·00	18·98	23·13	
25	..	2·06	3·47	5·30	7·80	10·57	14·18	17·90	22·05	
26	..	..	3·15	4·99	7·20	10·06	13·28	16·91	20·88	(ii) *Given.*—Girder as in (*i*) with web $\frac{7}{16}$ inch thick.
27	..	..	2·88	4·78	6·72	9·38	12·45	16·01	19·89	
28	..	..	2·70	4·41	6·30	8·78	11·63	15·18	18·72	*Required.*—Clear horizontal distance between vertical stiffeners.
29	..	..	2·57	4·10	5·94	8·24	11·03	14·27	17·91	
30	..	..	2·48	3·78	5·64	7·76	10·43	13·45	17·01	The shearing stress per foot is 6 tons as in (*i*) ; in the column of the table for $\frac{7}{16}$-inch plates, the nearest stress to 6 tons is 6·20 tons, which occurs on the line for a distance of 23 inches. The distance required is therefore about 23 inches.
31	..	..	..	3·52	5·22	7·29	9·83	12·71	16·20	
32	..	..	..	3·26	5·04	6·89	9·30	12·05	15·30	
33	..	..	..	3·15	4·74	6·62	8·85	11·47	14·93	
34	..	..	..	2·99	4·44	6·35	8·25	10·89	13·77	
35	..	..	..	2·89	4·14	6·01	7·88	10·40	13·14	
36	..	..	..	..	3·84	5·67	7·50	9·90	12·51	
38	..	..	..	..	3·54	5·00	6·98	8·83	11·43	
40	..	..	..	..	3·30	4·32	6·30	8·17	10·35	
42	..	..	..	..	..	4·05	5·70	7·67	9·45	
44	..	..	..	..	..	3·78	5·10	6·93	8·82	
46	..	..	..	..	..	..	4·58	6·27	8·28	
48	..	..	..	..	..	..	4·35	5·69	7·56	

TABLE IV.

Bending Movements, etc., of Beams in various Conditions and under various Arrangements of Loads.

TABLE

Bending moments, etc., of beams in various conditions

W = external load; *i.e.*, load exclusive of weight of beam .. .. } All measured by *same* unit of weight, *i.e.*, all in
E = Modulus of elasticity per square inch .. .. .. } tons, or all in lb., etc.
S = Limiting stress per square inch .. .. .. }

l = AB = { distance from centre to centre of bearings in all cases except IV and VIII .. .. } in *inches*.
x = AP or BP = { distance from centre of bearing at fixed end to free end in Cases IV and VIII .. .. } in *inches*.
= distance from a bearing to any point P .. .. }

C represents the centre of AB.

K, L represent the points of Contraflexure in beams fixed at both ends.

K represents the point of Contraflexure in beams fixed at one end and supported at the other end.

Q represents the point where maximum deflection occurs in cases where this point does not correspond with C.

I = Moment of inertia of section of beam in quartic inches.

d = depth of beam in inches.

Case.	Condition of beam.	Arrangement of load (dead in all cases).	Bending moments.		Shearing stress.	
			Maximum.	Elsewhere.	Maximum.	Elsewhere.
1	2	3	4	5	6	7
I	Supported at A and B.	Uniformly distributed.	$\frac{Wl}{8}$ at C.	$\frac{Wx\,(l-x)}{2l}$ at P; x = AP, if P lies in AC. x = BP, if P lies in BC.	$\frac{W}{2}$ at A and B.	$\frac{W\,(l-2x)}{2l}$ at P.
II	Fixed at A and B.	Do. do.	$\frac{Wl}{12}$ at A and B.	$\frac{Wl}{24}$ at C; $\frac{W\,(l^2-6lx+6x^2)}{12l}$ at P; x as in Case I.	Do. do.	Do. do.
III	Fixed at A, supported at B.	Do. do.	$\frac{Wl}{8}$ at A.	$\frac{Wl}{16}$ at C; $\frac{9Wl}{128}$ at D; AD = $\frac{5l}{8}$; BD = 8 = $\frac{BK}{2}$	$\frac{5W}{8}$ at A.	$\frac{3\,W}{8}$ at B. $\frac{5l-8x}{8l}$ at P. x = AP.
IV	Fixed at A, free at B.	Do. do.	$\frac{Wl}{2}$ at A.	$\frac{W\,(l-x)^2}{2l}$ at P; x = AP.	W at A.	$\frac{W\,(l-x)}{l}$ at P.
V	Supported at A and B.	Concentrated at C.	$\frac{Wl}{4}$ at C.	$\frac{Wx}{2}$ at P; x as in Case I.	$\frac{W}{2}$	$\frac{W}{2}$ except at C. *Nil* at C.
VI	Fixed at A and B.	Do. do.	$\frac{Wl}{8}$ at A, B, and C.	$\frac{W}{8}(l-4x)$ at P; x as in Case I.	Do.	Do. do.
VII	Fixed at A, supported at B.	Do. do.	$\frac{3Wl}{16}$ at A.	$\frac{5Wl}{32}$ at C.	$\frac{11W}{16}$ from A to C.	$\frac{5\,W}{16}$ from B to C. *Nil* at C.
VIII	Fixed at A, free at B.	Concentrated at B.	Wl at A.	$W\,(l-x)$ at P; x = AP.	W.	W.

IV.

and under various arrangements of loads.

y = distance in inches from Neutral Axis to extreme fibre of section ($=\frac{d}{2}$ in case of sections symmetrical above and below the Neutral Axis.)

$\frac{SI}{y}$ = Moment of resistance of any section.

$\frac{2\,SI}{d}$ = Moment of resistance of sections symmetrical above and below the Neutral Axis.

m or m_1 = deflection factor for given load W.

n or n_1 = deflection factor for limiting stress S.

Deflection, for uniform section (Cases I to XIV) = $m \times \frac{W}{EI}$, or $n \times \frac{S}{E}$.

Deflection, for uniform strength (Cases I, IV, V and VIII only) = $m_1 \times Wl^3\,\frac{Wl^3}{EI}$ or $n_1 \times \frac{Sl^2}{Ey}$.

Positions of points of Contraflexure.	Deflection factors. Uniform section. m.	Uniform section. n.	Uniform strength and depth. m_1.	Uniform strength and depth. n_1.	Uniform strength and breadth. m_1.	Uniform strength and breadth. n_1.	Case.
8	9	10	11	12	13	14	15
.....	$\frac{5l^3}{384}$ at C. $\frac{x\,(l^3-2lx^2+x^3)}{24l}$ at P; x as for bending moments.	$\frac{5l^2}{48y}$ at C. $\frac{x\,(l^3-2lx^2+x^3)}{8l^2y}$ at P.	$\frac{1}{8\sqrt{3}}$ at C.	$\frac{1}{8}$ at C.	·0178 at C.	·1426 at C.	I
AK = BL = ·211 × l	$\frac{l^3}{384}$ at C; $\frac{x^2\,(l-x)^2}{24l}$ at P; x as in Case I.	$\frac{l^2}{16d}$ at C. $\frac{x^2\,(l-x)^2}{l^2d}$ at P.	..	..	..	..	II
AK = $\frac{l}{4}$ BK = $\frac{3l}{4}$	$\frac{l^3}{192}$ at C; ·0054 × l^3 at Q; AQ = ·578l. $\frac{x^2\,(3l-2x)\,(l-x)}{48l}$ at P; x = AP	$\frac{l^2}{12d}$ at C. $\frac{·0864 \times l^2}{d}$ at Q. $\frac{x^2\,(3l-2x)\,(l-x)}{3l^2d}$ at P.	..	..	..	..	III
.....	$\frac{l^3}{8}$ at B.	$\frac{l^2}{4y}$ at B.	$\frac{1}{4}$ at B.	$\frac{1}{8}$ at B.	$\frac{1}{2}$ at B.	1 at B.	IV
.....	$\frac{l^3}{48}$ at C.	$\frac{l^2}{12y}$ at C.	$\frac{1}{8\sqrt{3}}$ at C.	$\frac{1}{8}$ at C.	$\frac{1}{4\sqrt{3}}$ at C.	$\frac{1}{6}$ at C.	V
AK = BL = $\frac{l}{4}$	$\frac{l^3}{192}$ at C.	$\frac{l^2}{12d}$ at C.	..	..	..	..	VI
AK = $\frac{3l}{11}$ BK = $\frac{8l}{11}$	·0091 × l^3 at C. ·0093 × l^3 at Q; AQ = ·553 × l	·097 $\frac{l^2}{d}$ at C. ·099 $\frac{l^2}{d}$ at Q.	..	..	..	..	VII
.....	$\frac{l^3}{3}$ at B.	$\frac{l^2}{3y}$ at B.	$\frac{1}{2}$ at B.	$\frac{1}{8}$ at C.	$\frac{2}{3}$ at B.	$\frac{2}{3}$ at C.	VIII

TABLE

Case.	Condition of beam.	Arrangement of load (dead in all cases).	Bending Moments.		Shearing
			Maximum.	Elsewhere.	Maximum.
1	2	3	4	5	6
IX	Supported at A and B.	Half$\left(\frac{W}{2}\right)$ concentrated at each of two points, D and F, equidistant from the centre. AD = BF = a BD = AF = b $b > a$	$\frac{Wa}{2}$ from D to F.	$\frac{Wx}{2}$ at any point P in AD or BF; x = AP, if P lies in AD; x = BP, if P lies in BF.	$\frac{W}{2}$ from A to D and from B to F.
X	Fixed at A and B.	Do. do.	$\frac{Wab}{2l}$ at A and B.	$\frac{Wa^2}{2l}$ from D to F.	Do. do.
XI	Fixed at A, supported at B.	Do. do.	$\frac{3Wab}{4l}$ at A.	$\frac{Wa(3b^2-2l^2)}{4l^2}$ $\frac{Wa(2l^2-3ab)}{4l^2}$ at F.	$\frac{W(2l^2+3ab)}{4l^2}$ from A to D.
XII	Supported at A and B.	Concentrated at any point D. AD = a. BD = b. $b > a$	$\frac{Wab}{l}$ at D.	$\frac{Wbx}{l}$ at any point P in AD $\frac{Wax}{l}$ at any point P in BD. x = AP, if P lies in AD. x = BP, if P lies in BD.	$\frac{Wb}{l}$ from A to D.
XIII	Fixed at A and B.	Do. do.	$\frac{Wab^2}{l^2}$ at A.	$\frac{2Wa^2b}{l^2}$ at B. $\frac{2Wa^2b^2}{l^3}$ at D.	$\frac{Wb^2(l+2a)}{l^3}$ from A to D.
XIV	Fixed at A, supported at B.	Concentrated at any point D. AD = a. BD = b.	Either $\frac{Wab(l+b)}{2l^2}$ at A (*i*) or $\frac{Wa^2b(2l+b)}{2l^3}$ at D (*ii*) or ·172 Wl at A and D (*iii*).	(*i*) is $\gtrless$ (*ii*) According as b is $\gtrless l(\sqrt{2}-1)$ or $0{\cdot}414 \times l$ *i.e.*, according as $2b^2 \gtrless a^2$ (*iii*) is the bending moment when $2b^2 = a^2$.	$\frac{Wb(3l^2-b^2)}{2l^3}$ from A to D.

Note (i).—The particulars given in this table for Cases II, III, VI, VII, X, XI, XIII and XIV apply only to beams angles or tees in their usual positions (*i.e.*, symmetrical axis not horizontal), nor to any sections of uniform strength. Design (Major G. K. Scott Moncrieff, R.E.).

Note (ii).—The deflection due to the external load + weight of beam = approximately the sum of the two deflections external load + the deflection at Q due to the weight of the beam; the latter deflection (in cases of beams of uniform for x in the deflection formula given under Case I for deflection at any given point.

Note (iii).—To find the maximum load which a given beam will carry:—Equate the maximum bending moment due $\frac{2SI}{d}$; hence $W = \frac{16S}{ld} - w$, where w = weight of beam.

Note (iv).—In the case of built up steel or iron girders, the moment of resistance = S d_1 N in which d_1 = effective

IV—*concluded.*

Stress.	Positions of points of Contraflexure.	Deflection factors.		Case.
		Uniform section.		
Elsewhere.		m.	n.	
7	8	9	10	11
Nil from D to F.	....	$\frac{a^2(3l^2-4a^2)}{48}$ at C.	$\frac{l^2-4a}{24y}$ at C.	IX
Do. do.	AK = BL = $\frac{ab}{l}$	$\frac{a^2(3l-4a)}{48}$ at C.	$\frac{al(3l-4a)}{12bd}$ at C.	X
$\frac{W(2l^2-3ab)}{4l^2}$ from B to F.	AK = $\frac{l(3b-2l)}{3b}$, BK = $\frac{2l^2}{3b}$ if $3b^2 > 2l^2$	If $b^3 > 2a^3$, Q lies in DF, and m = $\frac{a}{54b^2}\{l^4-9a^2(l^2+b^2)+l(3a^2+(l^2)^{\frac{3}{2}}\}$	$m \times \frac{8l}{3abd}$ in DF	XI
$\frac{3Wab}{4l^2}$ from D to F.	AK = a, BK = b if $3b^2 = 2l^2$	If $b^3 = 2a^3$, Q is at F, and $m = \frac{a^2b}{6}$	$\frac{4al}{9d}$ at F	
Nil at F.	AK = $\frac{3abl}{3ab+2l^2}$, BK = $\frac{2l^3}{3ab+2l^2}$ if $3b^2 > 2l^2$	If $b^3 > 2a^3$, Q lies in BF, and m = $\frac{abl}{6}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ . . .	$\frac{4l^2}{9d}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ in BF	
	....	$\frac{a^2b^2}{3l}$ at D	$\frac{ab}{3y}$ at D	XII
$\frac{Wa}{l}$ from B to D. *Nil* at D.		$\frac{a}{3l}\left(\frac{l^2-a^2}{3}\right)^{\frac{3}{2}}$ at Q; BQ = $\sqrt{\left(\frac{l^2-a^2}{3}\right)}$	$\frac{1}{3by}\left(\frac{l^2-a^2}{3}\right)^{\frac{3}{2}}$ at Q	
		$\frac{a^3b^3}{3l^3}$ at D	$\frac{2a^2b}{3d}$ at D	XIII
$\frac{Wa^2(l+2b)}{l^3}$ from B to D. *Nil* at D.	AK = $\frac{al}{l+2a}$, BL = $\frac{bl}{l+2b}$	$\frac{2a^2b^3}{3(l+2b)^2}$ at Q; BQ = $\frac{2bl}{l+2b}$ = 2BL	$\frac{4abl^2}{3d(l+2b)^2}$ at Q	
		$\frac{a^3b^2(3l+b)}{12l^3}$ at D	$\frac{ab(3l+b)}{3(2l+b)d}$ at D	XIV
$\frac{Wa^2(3l-a)}{2l^3}$ from B to D. *Nil* at D.	AK = $\frac{l(l^2-b^2)}{3l^2-b^2}$, BK = $\frac{2l^3}{3l^2-b^2}$	If $b > 0{\cdot}414 \times l$, $m = \frac{a^2b}{6}\sqrt{\left(\frac{b}{2l+b}\right)}$ at Q; BQ = $l\sqrt{\left(\frac{b}{2l+b}\right)}$ (*i*)	$\frac{2al^2}{3(l+b)d}\sqrt{\left(\frac{b}{2l+b}\right)}$ at Q (*i*)	
		If $b < 0{\cdot}414 \times l$, $m = \frac{b(l^2-b^2)^2}{3(3l^2-b^2)^2}$ at Q; BQ = $\frac{l(l^2+b^2)}{3l^2-b^2}$ (*ii*)	$\frac{4al^3(l+b)^3}{3(2l+b)(3l^2-b^2)^2d}$ at Q (*ii*)	
		If $b = 0{\cdot}414 \times l$, m = ·0098 × l^2 at D (*iii*)	·1144 $\frac{l^2}{d}$ at D (*iii*)	

of uniform section, *symmetrical about and below the neutral axis*; they do not apply to other beams of uniform section such as Those for Cases X, XI, XIII, and XIV have been deduced from the diagrams given in Part II, Principles of Structural

calculated separately, *e.g.*, in Case XII the total maximum deflection would probably be the deflection at Q due to the

section) could be found by substituting the value of BQ (Case XII) or of l — BQ (if BQ, Case XII, were greater than $\left(\frac{l}{2}\right)$

to the external load + that due to the weight of the beam, to the moment of resistance, *e.g.*, in Case III $\frac{Wl}{8} + \frac{wl}{8} =$

depth and N = net area of one flange after deducting rivet holes.

No. 2938 W., 6TH NOVEMBER 1899.

ORDER—No. 2938 W., dated 6th November 1899.

Communicated to officers of the Public Works Department for information and guidance in continuation of G.O., No. 480 W., dated 1st March 1895.

(True Extract.)

Under Secretary to Government.

To the Chief Engineer for Irrigation.
,, the Consulting Engineer for Railways.
,, all Superintending Engineers
,, the Consulting Architect to Government.
,, the Superintendent of Works, Lighthouse Division.

Exd. A. Rungasawmy Aiyangar.

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SUBJECT.

Endents for Rolled Steel Joists.

Forwarding a Statement Suggesting for the preparation of Indents

To
The Chief Engr. for Irrigation ✓
" Consg. Engr. for Railways ✓
All Supdg. Engrs. ✓
The Consg. Architect to Govt. ✓
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Circular No. 4 Railway.

GOVERNMENT OF INDIA.

PUBLIC WORKS DEPARTMENT.

RAILWAY STORES.

Simla, the 20th September 1899.

The Secretary to the Government of Madras, Public Works Department.
The Joint Secretary to the Government of Madras, Public Works Department, Railway Branch.
The Secretaries to the Government of Bombay, Public Works Department, General and Railway Branches.
The Secretaries to the Governments of Bengal, the North-Western Provinces and Oudh, and the Punjab, Public Works Department.
The Secretaries to the Government of Burma, Public Works Department, General and Railway Branches.
The Chief Commissioners of the Central Provinces, Assam, and Coorg.
The Resident at Hyderabad.
The Resident in Mysore.
The Agents to the Governor General for Central India and Rajputana, and Baluchistan.
The Accountant General, Public Works Department.
The Director General of Military Works.
The Consulting Engineers to the Government of India for Railways, Calcutta, Lucknow, and Assam.
The Director General of Telegraphs.
The Managers, North Western, Oudh and Rohilkhund, East Coast and Eastern Bengal State Railways.
The Engineer-in-Chief, Hardwar-Dehra Railway.
The Engineer-in-Chief, Indus Bridge Works and Shadipali-Balotra Railway.
The Engineer-in-Chief, Godavery Bridge Works.
The Engineer-in-Chief, Ghaziabad-Moradabad Railway.
The Port Storekeeper, State Railway Department, Calcutta.

The under-mentioned paper is forwarded to the Local Governments and Administrations and to the Officers noted on the margin for information and guidance, in supersession of Government of India Circular No. 2 Ry. of 25th January 1895.

2. These revised instructions are applicable to all Branches of the Public Works Department.

By order,

C. J. C.

Under Secretary.

Suggestions for the preparation in India of Indents for Rolled Steel Joists.

(*Revised December 1898.*)

1. Difficulty is sometimes experienced in procuring rolled joists of the exact sections demanded. It is therefore recommended that a detailed statement, giving limits of variation and other particulars for each item, should be furnished on the form shown on page 3.

2. Joists are tested with central loads and must show no permanent set after test. The loads are arranged so as to produce a limiting stress of 7½ tons per square inch. Samples cut from the joists are required to bear an ultimate tensile stress of 27 to 32 tons per square inch, with an elongation of 20 per cent. in a length of 8 inches.

3. Steel joists are now invariably supplied, because "open hearth" mild steel has practically supplanted wrought-iron for structural work.

4. Taking the modulus of elasticity for steel at 12,500 * tons per square inch, and that for iron at 11,125 † tons per square inch, the deflection of a steel joist with a limiting stress of 5·62 tons per square inch is the same as that of an iron joist of the same section, and under the same conditions, with a limiting stress of 5 tons. With higher limits of stress, as 7½ tons, the deflection is, of course, increased.

5. Table I, annexed, gives particulars of various sections, together with constants for each, from which loads and deflections can be easily ascertained. The sections enumerated are

* Professor T. A. Hearson, M.I.C.E. | † Molesworth's Pocket Book.

To The Secretary to the Govt of Madras Public Works Deptt.
Sc will follow from Calcutta

all stated in published lists to be "stock" sections, and should therefore be readily obtainable, but special rolling and consequent delay are generally found necessary, probably because contractors will not cut stock lengths to waste.

6. In estimates of weight, 2 per cent. should be added to list weights of joists to allow for variations in rolling.

7. Tables II and III which follow the Appendix are intended to reduce clerical labour in cases where built-up plate girders have to be designed. Table IV gives particulars of bending moments, etc., of beams under various conditions.

Detail of Rolled Steel Joists demanded on Indent _____

Item of Indent.	Section required.			Maximum and minimum limits of dimensions of section which may be allowed in case the exact section cannot be procured.				Length.	Clear span.	Number required.			Dead load, including weight of joist.	* Arrangment of load.	† Condition of joist.	Limiting stress per square inch.	Remarks.
				Depth.		Width.				In dead lengths, *i.e.*, both ends machined.	With one end machined.	With neither end machined.					State whether built-up plate girders may be substituted for joists, and, if so, whether they must be of uniform *section* throughout or may be of uniform *strength and depth*.
	Depth.	Width.	Weight per foot run.	Max.	Min.	Max.	Min.										
	In.	In.	Lb.	In.	In.	In.	In.	Ft. in.	Ft. in.				Tons.			Tons.	

* *i.e.*, Distributed.
Central.
etc.

† *i.e.*, Fixed both ends.
Supported, ditto.
etc.

TABLE I.

Rolled Steel Joists.

Particulars of Sections, and Constants for Load and Deflection, with a limiting Stress of 7·5 tons per square inch.

Case I.

Condition of Joist . . . Supported at both ends.

Arrangement of Load . . Dead, uniformly distributed, weight of joist included.

Section.	Weight per lineal foot.	Thickness of web.	Moment of inertia.	Constants.	
				For load.	For deflection.
Inches.	Lbs.	Inch.	Quartic inches.		
20 × 7½	89	·60	1,646·6	823·3	·09
19¾ × 7¼	96	11/16	1,715·48	868·60	·091
18 × 7	75	·55	1,150·53	639·2	·10
17¾ × 6¾	80	·625	1,163·05	655·21	·101
16 × 6	62	·56	722·36	451·5	·1125
16 × 5	50	·51	556·01	347·50	·1125
15¾ × 6⅛	62	9/16	704·58	447·33	·114
15 × 6	59	·54	617·05	411·37	·12
15 × 5⅞	56·5	9/16	572·64	381·76	·12
15 × 5	42	·422	414·51	276·34	·12
14 × 6	57	·51	520·65	371·9	·1286
14 × 6	46	·435	422·57	301·83	·1286
13⅞ × 5½	46	7/16	419·63	299·73	·13
13 × 5	41·5	·51	300·65	231·27	·138
12 × 6	54	·51	369·91	308·27	·15
12 × 6	44	·41	314·4	262·00	·15
12 × 6	42	·42	296·02	246·68	·15
12 × 5	39	·44	254·48	212 07	·15
12 × 5	32	·35	220·95	184·13	·15
11 13/16 × 5	37	·42	242·46	205·25	·152
10 × 6	45	·489	215·97	215·97	·18
10 × 5	35	·48	159·97	159·97	·18
10 × 5	29	·35	141·67	141 67	·18
10 × 4½	30	·387	141·42	141·42	·18
9⅞ × 6	41	·42	200·23	202·76	·182
9⅞ × 5	27	·36	125·90	127·50	·182
9 13/16 × 4½	36	·516	155·22	158·19	·184
9 1/16 × 4	22	·33	80·54	88·87	·198
9 × 7	58	·777	216·81	240·90	·20
9 × 3¾	20	30	75·02	83·36	·20
8 × 6	35	·44	111·59	139·49	·225
8 × 6	32	7/16	101·25	126·56	·225
8 × 5	30	·40	93·86	117·33	·225
8 × 4	25	·41	73·24	91·55	·225
8 × 4	19	·329	56·48	70·60	·225
7⅞ × 5	27	·375	82·48	104·74	·2286
7⅞ × 4	19	5/16	56·16	71·31	·2286
7 × 3¾	18	·313	41·99	59·99	·257
7 × 3¾	16	·25	35·98	51·4	·257
7 × 3½	16	·25	38·80	55·43	·257
6¼ × 3½	18	·339	33·09	52·94	·288
6 1/16 × 2	9·5	·25	15·81	25·29	·288
6 1/16 × 5⅛	24	7/16	40·47	66·40	·294
6 × 5	25	·423	43·46	72·43	·30
6 × 4⅛	20	·434	33·35	55·58	·30
6 × 3⅛	13	·25	26·79	44·65	·30
6 × 3	16	·39	25·14	41·90	·30
6 × 3	13	·322	20·77	34·62	·30

Section.	Weight per lineal foot.	Thickness of web.	Moment of inertia.	Constants.	
				For load.	For deflection.
Inches.	Lbs.	Inch.	Quartic inches.		
6 × 2	12	·381	16·58	27·64	·30
5⅔ × 2	10·5	·329	12·9	23·45	·327
5¼ × 1½	9	·368	8·77	16·07	·327
5⅛ × 4½	17·5	5/16	23·14	45·15	·351
5 × 5	24	·371	29·55	59·10	·36
5 × 4½	22	·342	26·94	53·88	·36
5 × 4 3/16	19	·44	22·09	44·18	·36
5 × 3	15	·40	16·09	32·18	·36
5 × 3	11	·23	13·65	27·30	·36
5 × 3	10	·25	11·93	23·86	·36
5 × 2	9·5	·25	10·72	21·44	·36
4¾ × 1¾	10	·40	8·48	17·85	·379
4¾ × 1¾	6·5	3/16	6·50	13·68	·379
4 1/16 × 2 5/16	7·5	·21	7·67	16·36	·384
4⅝ × 3	14	·40	13·17	28·48	·39
4⅛ × 3	10	·23	8·42	20·41	·436
4 × 3	12	·299	9·03	22·58	·45
4 × 3	9·5	·225	7·538	18·85	·45
4 × 2	5·5	3/16	4·23	10·6	·45
4 × 1¾	8	·331	5·16	12·9	·45
4 × 1¾	5	·18	3·628	9·07	·45
3½ × 3	10·5	·35	5·87	16·77	·514
3½ × 1½	6	·296	2 91	8·31	·514
3⅛ × 1 5/8	4	·17	1·80	5·76	·576
3 1/16 × 3 1/16	8	·25	3·67	11·98	·588
3 × 3	10	·29	4·21	14·03	·60
3 × 1½	4·5	·156	1·90	6·33	·60
3 × 1¼	4	·218	1·47	4·90	·60

To use Table I.

(i) To find the load and corresponding deflection for a given section of joist with a given span :—

(*a*) Divide the constant for load by the distance *in feet* from centre to centre of bearings ;* the quotient is the total distributed load *in tons*, including the weight of the joist.†

(*b*) Multiply the constant for deflection by the square of the distance *in feet* from centre to centre of bearings,* and divide the result by 100 ; the quotient is the deflection in inches.

(ii) To select a joist for a given external load (*i.e.*, load exclusive of weight of joist) and given span :—

Multiply the load *in tons* by the distance *in feet* from centre to centre of bearings,* and take a section against which the constant for load is slightly higher than the result. The weight of the joist† should then be added to the external load, and the section tried as in (i) for load and deflection. The depth of a joist should not, as a rule, be less than from 1/18 to 1/20 span, and in the case of joists without lateral support, *e.g.*, the gantry girders for a traveller, the width should not be less than 1/30 span.

*† *See* notes at foot of page 6.

Cases II to XIII.

(iii) To ascertain loads, etc., in these cases, the constants in Table I must be multiplied by the factors given below for each particular case before proceeding as in (i) or (ii) above. In Cases IX to XIII the symbol L = distance in *feet* from centre to centre of bearings :—

Case.	Condition of joist.	Arrangement of load (dead in all cases).	Factors to be applied to	
			Load constant.	Deflection constant.
II	Fixed at both ends	Uniformly distributed	1·5	·3
III	Fixed at one end, supported at the other	Ditto	1	·415
IV	Fixed at one end, free at the other	Ditto	·25	2·4
V	Supported at both ends	Concentrated at centre	·5	·8
VI	Fixed at both ends	Ditto	1	·4
VII	Fixed at one end, supported at the other	Ditto	$\frac{2}{3}$	·475
VIII	Fixed at one end, free at the other	Concentrated at free end	·125	3·2
IX	Supported at both ends	Half ($\frac{W}{2}$) concentrated at each of two points equidistant from the centre; each point A feet from the nearer bearing, and B feet from the further bearing. (A + B = L)	$\frac{L}{4A}$	$\frac{6L^2-8A^2}{5L^2}$
X	Fixed at both ends		$\frac{L^2}{4AB}$	$\frac{6LA-8A^2}{5LB}$
XI	Fixed at one end, supported at the other		$\frac{L^2}{6AB}$	‡
XII	Supported at both ends	Concentrated at one point, A feet from the nearer, and B feet from the further bearing. (A + B = L).	$\frac{L^2}{8AB}$	$\frac{16AB}{5L^2}$
XIII	Fixed at both ends		$\frac{L^3}{8AB^2}$	$\frac{16A^2B}{5L^3}$

* In Cases IV and VIII substitute "centre of bearing of fixed end to free end" for "centre to centre of bearings."

† In Cases V and VIII, VI and VII, IX, X and XI, XII, XIII:

$$\text{Total load} = \text{external load} + \left\{ \begin{array}{l} \frac{1}{2} \\ \frac{2}{3} \\ \frac{L}{4A} \\ \frac{L^2}{6AB} \\ \frac{L^2}{8AB} \\ \frac{L^3}{12AB^2} \end{array} \right\} \times \text{weight of joist.}$$

In these cases the actual weight of the joist is not, for purposes of calculation, added to the external load to make up the total load, because the calculations would be made for concentrated loads, whereas the weight of the joist would be uniformly distributed. The calculated deflections will, in these cases, not be quite correct; and, in Cases XII and XIII, they will represent the deflection at the point where the load is applied, and not the maximum deflection. Nearer approximations may, however, be obtained by using the deflection formulæ given in Table IV, as explained in Note (ii) at the foot of pages 14 and 15.

‡ This factor is omitted as it cannot be stated shortly. *See* Case XI in Table IV.

Notes to Table I.

(*a*) The constants given in the table can readily be adapted to suit other limits of stress than that which has been used (7·5 tons per square inch), *e.g.*, if it be desired to limit the stress in a joist to 6 tons per square inch, the constants must be multiplied by $\frac{6}{7·5}$ or 0·8 before the load and deflection are determined.

(*b*) If the deflection calculated from the table be greater than can be allowed, and if the circumstances do not permit the use of a deeper section or of a heavier section of the same depth, the load calculated from the table must be reduced in the proportion of the allowable to the calculated deflection and the number of joists increased, *e.g.*, if the allowable deflection be 0·5 inch and the calculated deflection 0·6 inch, the load should be reduced to $\frac{5}{6}$ths of the calculated load; the stress would, of course, be reduced in the same proportion.

(*c*) If the deflection be limited to $\frac{1}{40}$th of an inch per foot run of the joist, the minimum depth (with the limit of stress used in the table) should be as follows for Case I, *viz.*, ·72 inch for each foot of distance between centres of bearings; *e.g.*, if the distance from centre to centre of bearings were 10 feet and the deflection were limited to $\frac{10}{40}$ or $\frac{1}{4}$ inch, the minimum depth of joist, Case I, should be 7·2 inches.

APPENDIX.

The constants in Table I have been calculated from the following data :—

Symbols.

W = Total distributed load in *tons*, including the weight of the joist.
L = Distance in *feet* from centre to centre of bearings.
S = Limiting stress in *tons* per square inch (7½ for mild steel).
d = Depth of joist in *inches*.
V = Deflection in *inches*.
E = Modulus of elasticity in *tons* per square inch (12,500 for mild steel).
*I = Moment of inertia of section of joist (from dimensions in inches).

Formulæ.

For *distributed* loads, ends of joists *supported* (Case I),—

$\frac{1}{8}$ WL × 12 = $\frac{SI \times 2}{d}$.

or WL = $\frac{4SI}{3d}$ (1) which is constant for each section for any fixed value of S.

$V = \frac{5}{48} \times \frac{SL^2 \times 12^2 \times 2}{Ed}$ (2).

From (2) $\frac{V}{L} = \frac{30S}{dE}$ (3) which is constant for each *depth* of joist for any fixed values of S and E.

For steel S = 7·5. E = 12500.

From (1) WL = $\frac{10I}{d}$. (Tabular constant for load.)

From (3) $\frac{\dagger 100V}{L^2} = \frac{1{\cdot}8}{d}$. (Tabular constant for deflection.)

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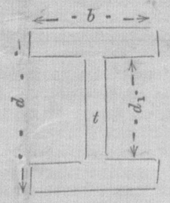
* When the thickness of the web is known as well as the depth, breadth, and weight per foot lineal, the moment of inertia for the section may be very approximately determined as follows :—

Let A = area of the section in square inches.

,, w = weight of the section in lbs. per foot lineal.

Then for steel A = $\frac{w}{3{\cdot}4}$, because 1 foot lineal of steel 1″ × 1″ weighs 3·4 lbs.

Assume the flanges to be of equal thickness throughout, *i.e.*, that the section is rectangular, as shown in figure,—



Then $I = \frac{1}{12}\left\{ bd^3 - (b-t)\, d_1^3 \right\}$, in which d_1 is the only dimension that is not given.

From the figure, A = $bd - (b-t)\, d_1$

whence $d_1 = \frac{bd - A}{b-t}$

and as A = $\frac{w}{3{\cdot}4}$; d_1 can be found and the value of I determined.

In manufacturers' lists of joists, it is generally stated that joists can be rolled about 10 per cent. heavier than list weights; the extra weight in such cases is obtained by slightly opening the rolls and thus thickening the web, and the consequent addition to the tabular constant for load is $\frac{xd}{4{\cdot}08}$ for a steel joist, x being the extra weight in lb. per foot lineal of the joist and d its depth in inches. The calculation is as follows :—

Let C = increase in area (in square inches) due to increase of x lb., per foot lineal, in weight.

Then $\frac{C}{d}$ = increase in thickness of web.

For the rectangle $d \times \frac{C}{d}$ $\begin{cases} I = \frac{1}{12} d^3 \times \frac{C}{d} = \frac{Cd^2}{12}. \\ \text{and from equation (1) above, } WL = \frac{4SI}{3d} = \frac{SCd}{9}. \end{cases}$

For steel, S = 7·5, C = $\frac{x}{3{\cdot}4}$, and WL = $\frac{xd}{4{\cdot}08}$ or approximately $\frac{xd}{4}$.

† The object of multiplying by 100 is to save long decimals in the tabular constants for deflection.

TABLE II.

Safe Resistances of Mild Steel Rivets (for Girders) to Shearing and Crushing.

Limiting Stresses { Shearing, 5½ tons per square inch of section.
Crushing, 8 tons per square inch of bearing area.

If d = diameter of rivet, and t = thickness of plate ; then Bearing Area = dt.

Diameter of rivet.	Resistance to Shearing.		Resistance to Crushing. Thickness of plates through which rivet passes—in inches.											Diameter of rivets.	Weight per 100 of rivet heads—snap or cup.
	Single shear.	Double shear.	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	13/16	I		
Inch.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Inch.	Lbs.
⅜	·61	1·21	·75	·94	1·13	...	...	...	...	...	...	...	...	⅜	1·47
7/16	·83	1·65	·88	1·10	1·31	1·53	...	...	...	...	...	...	...	7/16	2·34
½	1·08	2·16	1·00	1·25	1·50	1·75	2·00	...	...	...	...	...	...	½	3·48
9/16	1·37	2·73	1·13	1·41	1·69	1·97	2·25	2·53	...	...	...	...	...	9/16	4·95
⅝	1·69	3·37	1·25	1·56	1·88	2·19	2·50	2·81	3·13	...	...	...	...	⅝	6·79
11/16	2·04	4·08	1·38	1·72	2·06	2·41	2·75	3·09	3·44	3·78	...	...	...	11/16	9·04
¾	2·43	4·86	1·50	1·88	2·25	2·63	3·00	3·38	3·75	4·13	4·50	...	...	¾	11·74
13/16	2·85	5·70	1·63	2·03	2·44	2·84	3·25	3·66	4·06	4·47	4·88	5·28	...	13/16	14·93
⅞	3·31	6·61	1·75	2·19	2·63	3·06	3·50	3·94	4·38	4·81	5·25	5·69	6·13	⅞	18·63
15/16	3·79	7·59	1·88	2·34	2·81	3·28	3·75	4·22	4·69	5·16	5·63	6·09	6·56	15/16	22·92
1	4·32	8·64	2·00	2·50	3·00	3·50	4·00	4·50	5·00	5·50	6·00	6·50	7·00	1	27·82
1 1/16	4·88	9·75	2·13	2·66	3·19	3·72	4·25	4·78	5·31	5·84	6·38	6·91	7·44	1 1/16	33·37
1 ⅛	5·47	10·93	2·25	2·81	3·38	3·94	4·50	5·06	5·63	6·19	6·75	7·31	7·88	1 ⅛	39·61

Example of the use of this table :—

Given, { Thickness of web of girder—⅞ inch.
Pitch of rivets—4 inches ; rivets in double shear.
Shearing stress—9 tons per foot of depth, or per foot run.
Crushing stress—9 tons per foot of depth, or per foot run.

Required,—The diameter of the rivets necessary to resist crushing and shearing.

As there are three rivets per foot, the resistance per rivet must not be less than 3 or 3 tons : in the column for ⅞-inch plates, 3·06 tons, the resistance to crushing next above 3 tons is found in the line for ⅞ inch rivets ; the diameter required is therefore ⅞ inch. As the rivets are in double shear, their resistance to shearing is more than sufficient.

TABLE III.

Safe Resistances of Web Plates of Mild Steel Girders to Buckling. Calculated from the Formula of Professor T. C. Fidler, M.I.C.E., for columns of Mild Steel with Fixed Ends.

Clear unsupported distance between flange angles or between vertical stiffeners.	Thickness of web plate—in inches.								
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
Inches.	Safe shearing stress per foot in height of web—in tons.								
12	4·17	7·50	11·57	16·33	21·12	26·19	31·13	36·03	40·50
13	3·60	6·56	10·44	14·92	19·68	24·71	29·63	34·49	39·15
14	3·15	5·81	9·36	13·49	18·36	22·95	28·05	33·00	37·80
15	2·82	5·21	8·51	12·39	16·92	21·53	26·40	31·60	36·45
16	2·52	4·65	7·65	11·29	15·42	20·05	24·90	30·03	34·92
17	2·22	4·13	6·89	10·34	14·28	18·97	23·55	28·29	33·39
18	1·92	3·75	6·26	9·45	13·26	17·35	22·13	26·81	31·68
19	1·77	3·49	5·72	8·61	12·18	16·34	21·00	25·49	30·15
20	1·65	3·15	5·18	7·93	11·34	15·12	19·28	24·17	28·71
21	...	2·85	4·73	7·30	10·32	14·04	18·15	22·44	27·54
22	...	2·55	4·41	6·77	9·60	13·10	16·95	21·20	26·28
23	...	2·29	4·14	6·20	9·00	12·35	15·98	20·05	24·48
24	...	2·19	3·78	5·72	8·34	11·48	15·00	18·98	23·13
25	...	2·06	3·47	5·30	7·80	10·57	14·18	17·90	22·05
26	...	...	3·15	4·99	7·20	10·06	13·28	16·91	20·88
27	...	...	2·88	4·78	6·72	9·38	12·45	16·01	19·89
28	...	...	2·70	4·41	6·30	8·78	11·63	15·18	18·72
29	...	...	2·57	4·10	5·94	8·24	11·03	14·27	17·91
30	...	...	2·48	3·78	5·64	7·76	10·43	13·45	17·01
31	...	...	...	3·52	5·22	7·29	9·83	12·71	16·20
32	...	...	...	3·26	5·04	6·89	9·30	12·05	15·30
33	...	...	...	3·15	4·74	6·62	8·85	11·47	14·93
34	...	...	...	2·99	4·44	6·35	8·25	10·89	13·77
35	...	...	...	2·89	4·14	6·01	7·88	10·40	13·14
36	...	...	...	...	3·84	5·67	7·50	9·90	12·51
38	...	...	...	...	3·54	5·00	6·98	8·83	11·43
40	...	...	...	...	3·30	4·32	6·30	8·17	10·35
42	...	...	...	...	...	4·05	5·70	7·67	9·45
44	...	...	...	...	...	3·78	5·10	6·93	8·82
46	...	...	...	...	...	...	4·58	6·27	8·28
48	...	...	...	...	...	...	4·35	5·69	7·56

EXPLANATIONS.

Examples of the use of this table:—

(i) Given.—Depth of girder, 36 inches; vertical distance between flange angles, 28 inches; total shearing stress, 18 tons.
Required.—The thickness of the web to resist buckling when no stiffeners are used.
The shearing stress per foot in height of the web $= \frac{18}{3} = 6$ tons; in the table on the line for a clear distance of 28 inches between flange angles, the nearest stress to 6 tons is 6·30 tons, which occurs in the column for $\frac{1}{2}$-inch plates. The thickness required is therefore $\frac{1}{2}$ inch.

(ii) Given.—Girder as in (i) with web $\frac{7}{16}$ inch thick.
Required.—Clear horizontal distance between vertical stiffeners.
The shearing stress per foot is 6 tons as in (i); in the column of the table for $\frac{7}{16}$-inch plates, the nearest stress to 6 tons is 6·20 tons, which occurs on the line for a distance of 23 inches. The distance required is therefore about 23 inches.

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TABLE IV.

Bending Moments, etc., of Beams in various Conditions and under various Arrangements of Loads.

(*See pages* 12—15.)

TABLE

Bending Moments, etc., of Beams in various Conditions

W = external load ; *i.e.*, load exclusive of weight of beam . . }
E = Modulus of elasticity per sq. in. . . . . } All measured by *same* unit of weight, *i.e.*, all in tons, or all in lbs., etc.
S = Limiting stress per sq. in. . . . . }

l = AB = { distance from centre to centre of bearings in all cases except IV and VIII . . . . } in *inches*.
{ distance from centre of bearing at fixed end to free end in Cases IV and VIII . . . . }

x = AP or BP = distance from a bearing to any point P . . . . . .

C represents the centre of AB.
K, L represent the points of Contraflexure in beams fixed at both ends.
K represents the point of Contraflexure in beams fixed at one end and supported at the other end.
Q represents the point where maximum deflection occurs in cases where this point does not correspond with C.
I = Moment of inertia of section of beam in quartic inches.
d = depth of beam in inches.

Case.	Condition of beam.	Arrangement of load (dead in all cases).	Bending Moments.		Shearing Stress.	
			Maximum.	Elsewhere.	Maximum.	Elsewhere.
I	Supported at A and B.	Uniformly distributed.	$\frac{Wl}{8}$ at C	$\frac{Wx\,(l-x)}{2l}$ at P ; x = AP. if P lies in AC. x = BP., if P lies in BC.	$\frac{W}{2}$ at A and B	$\frac{W\,(l-2x)}{2l}$ at P
II	Fixed at A and B.	Do. do.	$\frac{Wl}{12}$ at A and B	$\frac{Wl}{24}$ at C ; $\frac{W\,(l^3-6lx^2+6x^3)}{12l}$ at P ; x as in Case I.	Do. do.	Do. do.
III	Fixed at A, supported at B.	Do. do.	$\frac{Wl}{8}$ at A	$\frac{Wl}{16}$ at C ; $\frac{9Wl}{128}$ at D ; AD = $\frac{5l}{8}$, BD = $8 = \frac{BK}{2}$	$\frac{5W}{8}$ at A	$\frac{3W}{8}$ at B ; $\frac{5l-8x}{8l}$ at P, x = AP
IV	Fixed at A, free at B.	Do. do.	$\frac{Wl}{2}$ at A	$\frac{W(l-x)^2}{2l}$ at P ; x = AP	W at A	$\frac{W\,(l-x)}{l}$ at P
V	Supported at A and B.	Concentrated at C.	$\frac{Wl}{4}$ at C	$\frac{Wx}{2}$ at P ; x as in Case I.	$\frac{W}{2}$	$\frac{W}{2}$ except at C *Nil* at C.
VI	Fixed at A and B.	Do. do.	$\frac{Wl}{8}$ at A, B, and C.	$\frac{W}{8}\,(l-4x)$ at P ; x as in Case I.	Do.	Do. do.
VII	Fixed at A, supported at B.	Do. do.	$\frac{3Wl}{16}$ at A	$\frac{5Wl}{32}$ at C	$\frac{11W}{16}$ from A to C	$\frac{5W}{16}$ from B to C. *Nil* at C.
VIII	Fixed at A, free at B.	Concentrated at B.	Wl at A	$W\,(l-x)$ at P ; x = AP	W	W

IV.

and under various arrangements of loads.

y = distance in inches from Neutral Axis to extreme fibre of section ($=\frac{d}{2}$ in case of sections symmetrical above and below the Neutral axis.)

$\frac{SI}{y}$ = Moment of resistance of any section.

$\frac{2SI}{d}$ = Moment of resistance of sections symmetrical above and below the Neutral Axis.

m or m_1 = deflection factor for given load W.

n or n_1 = deflection factor for limiting stress S.

Deflection, for uniform section (Cases I to XIV) = $m \times \frac{W}{EI}$, or $n \times \frac{S}{E.}$

Deflection, for uniform strength (Cases I. IV, V, and VIII only) = $m_1 \times Wl^3 \frac{W l^3}{EI}$ or $n_1 \times \frac{Sl^2}{Ey}$.

Positions of points of Contraflexure.	Deflection factors. Uniform section. m.	Uniform section. n.	Uniform strength and depth. m_1.	Uniform strength and depth. n_1.	Uniform strength and breadth. m_1.	Uniform strength and breadth. n_1.	Case
	$\frac{5l^3}{384}$ at C. $\frac{x\,(l^3-2lx^2+x^3)}{24l}$ at P ; x as for bending moments.	$\frac{5l^2}{48y}$ at C. $\frac{x\,(l^3-2lx^2+x^3)}{3l^2y}$ at P.	$\frac{1}{64}$ at C	$\frac{1}{8}$ at C	·0178 at C	·1426 at C	
AK = BL = ·211 × l	$\frac{l^3}{384}$ at C; $\frac{x^2\,(l-x)^2}{24l}$ at P ; x as in Case I.	$\frac{l^2}{16d}$ at C. $\frac{x^2\,(l-x)^2}{l^2d}$ at P.					II
AK = $\frac{l}{4}$ BK = $\frac{3l}{4}$	$\frac{l^3}{192}$ at C; ·0054 × l^3 at Q ; AQ = ·578l. $\frac{x^2\,(3l-2x)\,(l-x)}{48l}$ at P ; x = AP	$\frac{l^2}{12d}$ at C. $\frac{·0864\times l^2}{d}$ at Q. $\frac{x^2\,(3l-2x)\,(l-x)}{3l^2d}$ at P.					III
	$\frac{l^3}{8}$ at B	$\frac{l^2}{4y}$ at B	$\frac{1}{4}$ at B	$\frac{1}{8}$ at B	$\frac{1}{2}$ at B	1 at B	IV
	$\frac{l^3}{43}$ at C	$\frac{l^2}{11y}$ at C	$\frac{1}{32}$ at C	$\frac{1}{8}$ at C	$\frac{1}{24}$ at C	$\frac{1}{6}$ at C	V
AK = BL = $\frac{l}{4}$	$\frac{l^3}{193}$ at C	$\frac{l^2}{12d}$ at C					VI
AK = $\frac{3l}{11}$ BK = $\frac{8l}{11}$	·0091 × l^3 at C. ·0093 × l^3 at Q ; AQ = ·553 × l	·097 $\frac{l^2}{d}$ at C ·099 $\frac{l^2}{d}$ at Q					VII
	$\frac{l^3}{3}$ at B	$\frac{l^2}{3y}$ at B	$\frac{1}{2}$ at B	$\frac{1}{8}$ at C	$\frac{2}{3}$ at B	$\frac{2}{3}$ at C	VIII

[*Continued*

TABLE IV—*concluded.*

Case.	Condition of beam.	Arrangement of load (dead in all cases).	Bending Moments.		Shearing	Stress.	Positions of points of Contraflexure.	Deflection factors. Uniform section.		Case.
			Maximum.	Elsewhere.	Maximum.	Elsewhere.		m.	n.	
IX	Supported at A and B.	Half $\left(\frac{W}{2}\right)$ concentrated at each of two points, D and F, equidistant from the centre. AD = BF = a. BD = AF = b. $b > a$	$\frac{Wa}{2}$ from D to F	$\frac{Wx}{2}$ at any point P in AD or BF; x = AP, if P lies in AD; x = BF, if P lies in BF.	$\frac{W}{2}$ from A to D and from B to F	*Nil* from D to F		$\frac{a^2(3l^2-4a^2)}{48}$ at C	$\frac{l^3-4a}{24y}$ at C	IX
X	Fixed at A and B.	Do. do.	$\frac{Wab}{2l}$ at A and B	$\frac{Wa^2}{2l}$ from D to F.	Do. Do.	Do. do.	AK = BL = $\frac{ab}{l}$	$\frac{a^2(3l-4a)}{48}$ at C	$\frac{al(3l-4a)}{12bd}$ at C	X
XI	Fixed at A, supported at B	Do. do.	$\frac{3Wab}{4l}$ at A	$\frac{Wa(3l^2-2l^2)}{4l^2}$ $\frac{Wa(2l^2-3ab)}{4l^2}$ at F.	$\frac{W(2l^2+3ab)}{4l^2}$ from A to D.	$\frac{W(2l^2-3ab)}{4l^2}$ from B to F. $\frac{3Wab}{4l^2}$ from D to F. *Nil* at F.	AK = $\frac{l(3b-2l)}{3b}$, BK = $\frac{2l^2}{3b}$ if $3b^2 > 2l^2$; AK = a, BK = b if $3b^2 = 2l^2$; AK = $\frac{3abl}{3ab+2l^2}$, BK = $\frac{2l^2}{3ab+2l^2}$ if $3b^2 < 2l^2$	If $b^3 > 2a^3$, Q lies in DF, and $m = \frac{a}{54b^2}\{l^3-8a^2(l^2+b^2)+l.(3a^2+(l^2)^{\frac{3}{2}}\}$; If $b^3 = 2a^3$, Q is at F, and $m = \frac{a^2b}{6}$; If $b^3 < 2a^3$, Q lies in BF, and $m = \frac{abl}{6}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ . . .	$m \times \frac{8l}{3abd}$ in DF $\frac{4al}{9d}$ at F $\frac{4l^2}{9d}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ in BF	XI
XII	Supported at A and B.	Concentrated at any point D. AD = a. BD = b. $b > a$	$\frac{Wab}{l}$ at D	$\frac{Wbx}{l}$ at any point P in AD $\frac{Wax}{l}$ at any point P in BD. x = AP, if P lies in AD. x = BP, if P lies in BD.	$\frac{Wb}{l}$ from A to D	$\frac{Wa}{l}$ from B to D. *Nil* at D.		$\frac{a^2b^2}{3l}$ at D; $\frac{a}{3l}\left(\frac{l^2-a^2}{3}\right)^{\frac{1}{2}}$ at Q; BQ = $\sqrt{\left(\frac{l^2-a^2}{3}\right)}$	$\frac{ab}{3y}$ at D $\frac{1}{3by}\left(\frac{l^2-a^2}{3}\right)^{\frac{3}{2}}$ at Q	XII
XIII	Fixed at A, and B.	Do. do.	$\frac{Wab^2}{l^2}$ at A	$\frac{Wa^2b}{l^2}$ at B. $\frac{2Wa^2b^2}{l^3}$ at D	$\frac{Wl^2(l+2a)}{l^3}$ from A to D	$\frac{Wa^2(l+2b)}{l^3}$ from B to D. *Nil* at D.	AK = $\frac{al}{l+2a}$, BL = $\frac{bl}{l+2b}$	$\frac{a^2b^2}{3l^2}$ at D; $\frac{2a^2b^2}{3(l+2b)^2}$ at Q; BQ = $\frac{2bl}{l+2b} = 2BL$	$\frac{2a^2b}{3ld}$ at D $\frac{4abl^2}{3d(l+2b)^2}$ at Q	XIII
XIV	Fixed at A, supported at B.	Concentrated at any point D. AD = a. BD = b	Either $\frac{Wab(l+b)}{2l^2}$ at A (i) or $\frac{Wa^2b(2l+b)}{2l^3}$ at D (ii) or .172 Wl at A and D (iii)	(i) is $>=<$ (ii) According as b is $>=<l\left(\sqrt{2}-1\right)$ or 0·414×l *i.e.*, according as $2b^2>=<a^2$ (iii) is the bending moment when $2b^2=a^2$	$\frac{Wb(3l^2-b^2)}{2l^3}$ from A to D	$\frac{W(2l^2-3ab)}{4l^2}$ from B to F. $\frac{3Wab}{4l^2}$ from D to F. *Nil* at F.	AK = $\frac{l(3b-2l)}{3b}$, BK = $\frac{2l^2}{3b}$ if $3b^2 > 2l^2$; AK = a, BK = b if $3b^2 = 2l^2$; AK = $\frac{3abl}{3ab+2l^2}$, BK = $\frac{2l^2}{3ab+2l^2}$ if $3b^2 < 2l^2$	(i) If $b^3 > 2a^3$, Q lies in DF, and $m = \frac{a}{54b^2}\{l^3-8a^2(l^2+b^2)+l.(3a^2+(l^2)^{\frac{3}{2}}\}$; If $b^3 = 2a^3$, Q is at F, and $m = \frac{a^2b}{6}$; (ii) If $b^3 < 2a^3$, Q lies in BF, and $m = \frac{abl}{6}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ . . . ; (iii) If $b = 0{\cdot}414 \times l$, $m = {\cdot}0098 \times l^3$ at D	(i) $m \times \frac{8l}{3abd}$ in DF $\frac{4al}{9d}$ at F; (ii) $\frac{4l^2}{9d}\sqrt{\left(\frac{ab}{2l^2-3ab}\right)}$ in BF; (iii) $\frac{ab}{3y}$ at D $\frac{1}{3by}\left(\frac{l^2-a^2}{3}\right)^{\frac{3}{2}}$ at Q	XIV

NOTE (i).—The particulars given in this table for Cases II, III, VI, VII, X, XI, XIII, and XIV apply only to beams of uniform section, *symmetrical above and below the neutral axis*; they do not apply to other beams of uniform section such as angles or tees in their usual positions (*i.e.*, symmetrical axis not horizontal), nor to any sections of uniform strength. Those for Cases X, XI, XIII, and XIV have been deduced from the diagrams given in Part II, Principles of Structural Design (Major G. K. Scott Moncrieff, R.E.).

NOTE (ii).—The deflection due to the external load + weight of beam = approximately the sum of the two deflections calculated separately, *e.g.*, in Case XII the total maximum deflection would probably be the deflection at Q due to the external load + the deflection at Q due to the weight of the beam; the latter deflection (in cases of beams of uniform section) could be found by substituting the value of BQ (Case XII) or of $l - $ BQ (if BQ, Case XII, were greater than $\left(\frac{l}{2}\right)$ for x in the deflection formula given under Case I for deflection at any given point.

NOTE (iii).—To find the maximum load which a given beam will carry:—Equate the maximum bending moment due to the external load + that due to the weight of the beam, to the moment of resistance, *e.g.*, in Case III $\frac{Wl}{8} + \frac{wl}{8} = \frac{2SI}{d}$; hence $W = \frac{16S}{ld} - w$, where w = weight of beam.

NOTE (iv).—In the case of built up steel or iron girders, the moment of resistance = S d_1 N in which d_1 = effective depth and N = net area of one flange after deducting rivet holes.

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