

Home (1918) Department.

Education

Ordinary SERIES.

G.O. No. 139

DATED 7. 2. 18

INDEXED

[Despatch Abstract.]

Indian Educational Service.—

Informing M.R.Ry. T. ~~K~~ Chinmayanandam that he should apply to the Government of India through the Government of Bengal for the inclusion of his name in the list of candidates suitable for the appointment to the ————

Indian Association for the Cultivation of Science,
210, Bow-Bazar Street.

Calcutta, 30th December 1917.



The Secretary to the Government of Madras,
Education Department.

Sir,

I have the honour to forward herewith for favourable consideration, an application (with enclosures) from Mr. T. K. Chinmayanandam, who comes from Madras, and is now successfully engaged in original research in this institution.

It will be evident from the enclosures that Mr. Chinmayanandam has had an exceptionally distinguished record in the Madras University, a record which impressed the authorities of the Indian Association for the Cultivation of Science so greatly that they awarded him a special scholarship tenable at Calcutta for prosecuting research in its physical laboratory.

I am in a position from personal knowledge to write in terms of the highest praise regarding Mr. Chinmayanandam's skill as an experimenter, the methodical orderliness of his work, his clarity and soundness of judgment, and the facility with which he has made himself familiar with the latest developments in theoretical physics. These are the qualities indispensable for a teacher of Science in advanced work, and I can with great confidence recommend him for an appointment in the Indian Educational Service.

The two research papers, copies of which accompany this letter, furnish evidence of the originality and scientific abilities

Calcutta, 19

of their author. Even a casual perusal of the paper on the "Flow of Energy in the Electro-magnetic Field surrounding a perfectly reflecting Cylinder" will show that Mr. Chinmayanandam has succeeded in developing and extending the work of some of the eminent physicists of the day, among whom may be mentioned Prof. R.W. Wood F. R. S. of Johns Hopkins University, and Prof. Debye of Munich, and that he has verified his theoretical results by an exhaustive and painstaking experimental study. The other paper on Resonance Radiation and the Quantum Theory deals with the theories of Atomic Structure which are at present engaging the attention of the leading physicists all over the world. Among the most remarkable results obtained in this paper may be mentioned the proof given (pages 5-7) that Dr. Silberstein's theory of Resonance Radiation is untenable, and the explanation suggested (page 25) of Barkla and Shearers' experimental result regarding the Velocity of Secondary Corpuscular Radiation. Another interesting paper on the Diffraction of Light by an Obliquely held Cylinder by Mr. Chinmayanandam is also shortly to be published.

I have the honour to be

Sir,

your most obedient servant

C. V. Raman

Sir Taraknath Palit Professor of Physics.

Calcutta University.

3/4
210, Bow. Bazar Street,
Calcutta.

To

The Secretary to

to the Government of Madras

Education Department.

Sir,

I understand that some names are to be sent in shortly, by the Madras Government to the Government of India, for appointment in the Indian Educational Service, and I have been advised to submit my application for one of those appointments through you.

My qualifications are briefly these. I passed the examination of the Madras University, held in 1916, entitling me to the degree of M.A. (Hons.) in Physics. I took a first class in that examination and was placed first in the list; and as will be evident from my university mark-sheets, copies of which I submit with this application, I have secured about 75 per cent. of the maximum number of marks I have, since passing the examination, gone through a course of training at the Teachers' College, Saidapet and am now a first Class Licentiate in Teaching.

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After the training course, I have taken up research work in Physics at the Indian Association for the Cultivation of Science, Calcutta, where I am now a Research Scholar. Since coming here, I have been able to do successful research work in three different subjects. Two of these are already published, and copies of the papers are enclosed herewith; the third will soon be published. The papers will soon be reprinted in the London Philosophical Magazine.

I have enclosed a testimonial from Mr. R. L. Jones, Professor of Physics at the Madras Presidency College, under whom I studied during my Honours course. Reference may also be made, if necessary, to Mr. H. S. Duncan, Principal of the Teachers' College, Saidapet.

As to the respectability of my family, I may mention that one of my brothers, Mr. T. K. Rajagopal is Assistant Comptroller General, Delhi. and another brother, Mr. T. K. Venkataraman is Lecturer in mathematics at the Teachers' College, Saidapet.

I am at present 22 years of age.

I remain,
Sir,

your most obedient servant
J. K. Chinmayanandam

Dated. 30th Dec. 1917 }

True copy.

Mr. T. K. Chinmayanandam was a student in my Honours
Classes at the Presidency College from July 1913 to April 1916.
At the end of this period he passed his degree examination
taking a First class in Honours in Physics, and the First Place
in that class. His whole university career is a record of
first classes, and I may say the same of his work with me
throughout his course. His abilities are of a high order, as
this record shows. I shall be most glad to answer any enquiries
about him.

3rd July, 1916.

(Sd.) R. L. Jones,
Prof. of Physics.

True copy. 6/ 7

Office of the Registrar, University of
Madras.

Senate House.

Dated 14 Jun 1916

OFFICIAL MEMORANDUM No. 4234

Ref. -- His communication dated 11-6-1916

The following marks were awarded to candidate No.29
(T.K.Chinmayanandam) at the B.A.(Honours) Degree Examination
of 1916 (Branch II-A)

Subjects	Number of marks awarded	Maximum number of marks.
Written Examination in the Main Subject	465	600
Practical Examination & Labora- tory Note Book in the Main Subject	445	600
Subsidiary Subject	114	200

(Sd.) John J. Hensman,
for Registrar.

True copy. 7/

Office of the Registrar, University of
Madras.

Senate House.

Dated 21 Jun 1916

OFFICIAL MEMORANDUM No. 4472

Ref. -- His communication dated 20-6-'16

The following marks were awarded to him at the Intermediate
Examination of 1913 (Reg. No. 123)

Subjects	Number of marks awarded	Maximum number of marks.
English	176	300
Tamil composition	40	100
Mathematics	70	100
Physics	72	100
Chemistry	77	100

(Sd.) Francis Dewsbury,
Registrar,

To

Mr. T. K. Chinmayanandam



To The D.P.I. (with current & enclosed in original to be returned with reply)

8/ 9 Homel Edu Dept
C No 4 A

memo No 4 A-1 D. 11-1-18
Referred to the D.P.I.
remarks

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5-18.

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11-1-18

resp'd
with encs
11-1-18

HOME (Education) DEPARTMENT.

Received 191 . Registered 20 JAN 1918 191

Subject.

Returning with remarks the Government memorandum No. 4 A-1 dated 11th January 1918 regarding the application of Mr. T.K. Chinmayanandam for an appointment in the Indian Educational Service.

ENCLOSURES.

No.

Ref. on Cur. No. 413/18

Spare copies

Station Madras, Dated the 18 January 1918

From (Name) The Hon Mr. J.H. Stone, M.A., F.R.H.S., C.I.E

(Designation) Director of Public Instruction

To The Secretary to the Government of Madras,

HOME (Education) Department.

Sir,

In returning herewith the papers referred to me for remarks with memorandum No. 4 A-1 dated 11th January 1918, I have the honour to state that in reply to a similar communication addressed to me I have informed M.R.Ry. C.V. Raman that M.R.Ry. Chinmayanandam should with reference to G.O.No. 928 Edl., dated 18th October 1912 make an application to the Government of India in the prescribed form through the Government of Bengal and that there is no vacancy in this Presidency for which Chinmayanandam's

attainments

60
attainments and aptitudes would specially fit him.

I have the honour to be,

Sir,

Your most obedient servant,

G. V. Srinivasan

for Director of Public Instruction.

He
18.1.18.
S.V.

11
The Indian Association for the Cultivation of Science.

On the Flow of Energy in the Electro-
magnetic Field surrounding a Per-
fectly Reflecting Cylinder.

By T. K. CHINMAYANANDAM, B.A. (HONS.),
*Research Scholar in the Indian Association for the Cultivation
of Science, Calcutta.*

(Reprinted from the Proceedings, Vol. III, Part V, 1917)

Calcutta :
1917.



**On the Flow of Energy in the Electro-
magnetic Field surrounding a
Perfectly Reflecting
Cylinder.**

By T. K. CHINMAYANANDAM, B.A. (HONS.),
*Research Scholar in the Indian Association for
the Cultivation of Science, Calcutta.*

§ 1. *Introduction.*

In a paper recently contributed to this journal,* Mr. N. Basu has discussed the general features of the phenomena observed in the immediate neighbourhood of a perfectly reflecting cylinder, on which plane light waves are incident in a direction at right angles to its axis. Further investigation was, however, necessary in order to establish the formulæ for the distribution of light intensity in the various parts of the field. These formulæ have now been obtained, and subjected to a detailed experimental test. Besides describing the results of a photometric study that has been carried out, the present paper also deals with the form of the

* N. Basu.—*On the Diffraction of Light by Cylinders of large Radius*, Proc. of the Ind. Association, Vol. III, part 3, 1917.

lines of flow of energy through the field, which, it is thought, may prove of interest with reference to the work of Profs. R. W. Wood* and Max Mason† on the simpler case of the interference field due to two point sources of light.

It may be remarked here that the phenomena, which form the subject of this paper may be strikingly shown on a large scale, without the aid of a microscope, by using a cylindrical surface of very large radius as the diffracting "edge." A strip of thick plate glass, two inches wide and about a yard long, may be bent into a circle of some yards radius by resting it on supports near the two ends, and loading the latter sufficiently. A slit illuminated by a Cooper-Hewitt lamp and placed at some distance from the surface in a line with it, may be used as the source of light. A very large number of fringes may then be seen with a low-power eye-piece, if the plane of observation be within a few feet of the cylindrical "edge." At greater distances, the fringes widen out; their visibility and number decrease, and their spacing alters with increasing distances from the cylinder in such manner as to approximate more and more closely to that of the diffraction fringes due to a straight edge. Some photographs which have been taken with the arrangement described above are shown in Plate VI, where figures (a), (b), and (c) correspond to the phenomena in planes at increasing distances from the edge.

* R. W. Wood.—*On the Flow of Energy in a System of Interference Fringes*, Phil. Mag., 18, p. 250.

† Max Mason.—*The Flow of Energy in an Interference Field*, Phil. Mag., 20, p. 290.

§ 2. The Form of the Illumination Curves.

Debye* has shown from the electro-magnetic theory, that at a great distance from a cylinder (assumed to be of large radius) on which plane waves are incident, the disturbance due to it is practically the same as that to be expected from the principles of geometrical optics, this statement however not being taken as correct in respect of points lying in a direction very nearly the same as that of the incident rays. Debye's results suggest a simple method of finding the distribution of intensity at points lying within the region of light, in the immediate neighbourhood of the cylinder. Let AOB (Fig. 1) represent the section of the

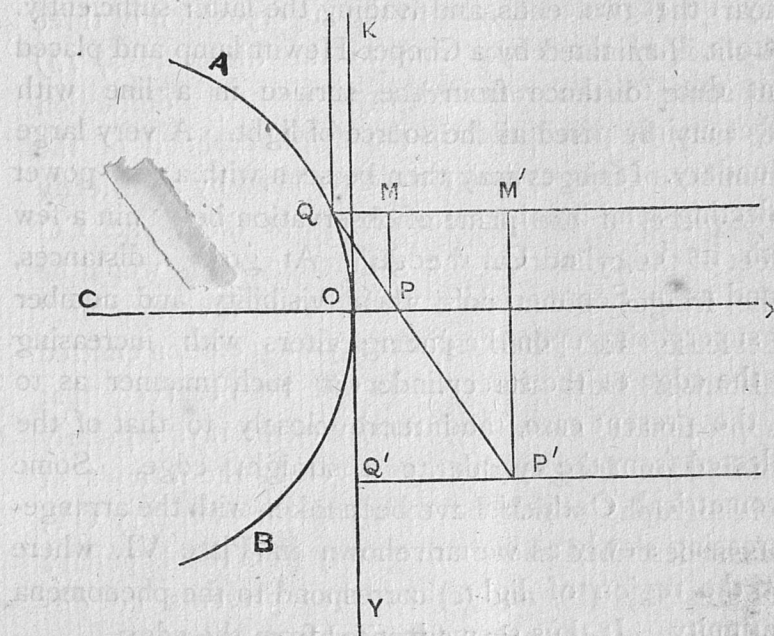


Fig. 1

* Debye.—Phys. Zeitschr., 9, pp. 775-778, Nov., 1908; also Science Abstracts, 1909, p. 88.

cylinder, and KOY the direction of the incident rays. We may assume that the fringes observed in the plane OX containing the edge of the cylinder grazed by the rays, are due solely to the interference of these rays with those reflected from the surface of the cylinder at varying angles. This would also be the case as regards any plane such as QR in advance of the edge. But the phenomena in a plane such as Q'P' which lies on the remote side of the edge, would not admit of such simple treatment, especially when we consider the effect at points lying not far from the boundary OY of the direct and reflected rays. In such a plane, the intensity at any point on the right of the boundary may be regarded as due to the superposition of three factors, (a) the effect due to the direct rays, (b) that due to the reflected rays, and (c) a diffraction effect mainly perceptible in the neighbourhood of the boundary. If the cylinder were replaced by a perfectly reflecting semi-infinite screen lying in the plane CO with its edge at O, the diffraction effect would be found as in Sommerfeld's well-known investigation,* by superposing upon the direct rays, a radiation emitted by the edge of the screen. It will be observed that in the present case, the intensity of the rays regularly reflected from the cylinder, as given by the formulæ of Geometrical Optics, is zero along the boundary OY, increasing slowly as we move away from the boundary into the region of light, and thus presents no discontinuity. It thus seems justifiable to assume that so far as regards the phenomena in the region on the

* Sommerfeld.—*On the Math. Theory of Diffraction*, Math. Annalen, Vol. XLVII, p. 317, 1895.

right-hand side of the boundary, the diffraction effect (c) is practically the same as in the case of a semi-infinite screen with its edge at O.

The distribution of intensity in the field may be readily found on the foregoing assumptions. Take the edge O as origin of co-ordinates, and rectangular axes OX, OY, perpendicular and parallel respectively to the direction of the incident rays, and let the angle OCQ, assumed to be small, be denoted by θ . The path difference between the direct and the reflected rays reaching any point P is

$$\begin{aligned}\delta &= 2\pi/\lambda (QP - MP) + \pi \\ &= 2k\theta^2 (y + a\theta) + \pi \text{ approximately,}\end{aligned}$$

if k be written for $2\pi/\lambda$.

Again, if the amplitude of the incident light be taken as unity, that of the reflected light may be written as

$$\left(\frac{\rho}{\rho + QP}\right)^{\frac{1}{2}}$$

where ρ is the radius of curvature of the reflected wave on emergence at Q. Since

$$\rho = \frac{a\theta}{2} \text{ approximately}$$

$$\frac{\rho}{\rho + QP} = \frac{a\theta}{2y + 3a\theta}$$

In any plane in advance of that passing through the edge of the cylinder ($y = -d$, say), the expression for the intensity of illumination at any point is

$$I = 1 + s - 2\sqrt{s} \cos \phi \quad \dots \quad (1)$$

where $\phi = 2k\theta^2 (y + a\theta)$

$$\text{and } s = \frac{a\theta}{3a\theta + 2y}.$$

The positions of the maxima and minima of illumination are given by

$$\frac{dI}{d\theta} = \frac{ds}{d\theta} \left(1 - \frac{\cos\phi}{\sqrt{s}}\right) + 2\sqrt{s} \sin\phi \frac{d\phi}{d\theta} = 0. \quad \dots (2)$$

Since $\frac{d\phi}{d\theta} = 4k\theta(y+a\theta) + 2ka\theta^2$, and contains the factor $2\pi/\lambda$, it will be large, so that the first term in equation (2) is negligible. That equation can hence be written

$$\sin\phi = 0. \quad \text{or} \quad 2\theta^2(y+a\theta) = m\lambda/2 \quad \dots (3)$$

while the relation between θ and x is given by

$$x = (y+a\theta)2\theta - a\theta^2/2 \\ = 2y\theta + 3a\theta^3/2. \quad \dots (4)$$

From (1) and (3) it is seen that the intensities of the successive maxima and minima are respectively proportional to

$$I_{\max.} = \left\{1 + \left(\frac{a\theta}{3a\theta - 2d}\right)^{\frac{1}{2}}\right\}^2 \\ I_{\min.} = \left\{1 - \left(\frac{a\theta}{3a\theta - 2d}\right)^{\frac{1}{2}}\right\}^2$$

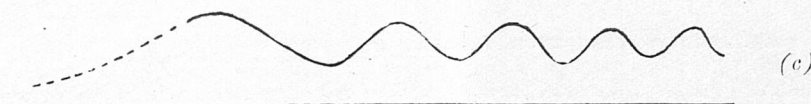
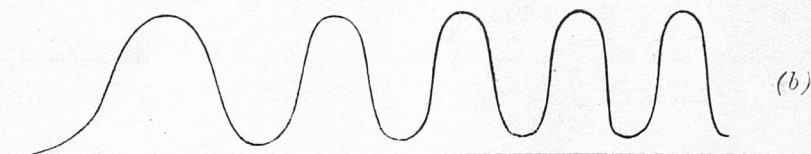
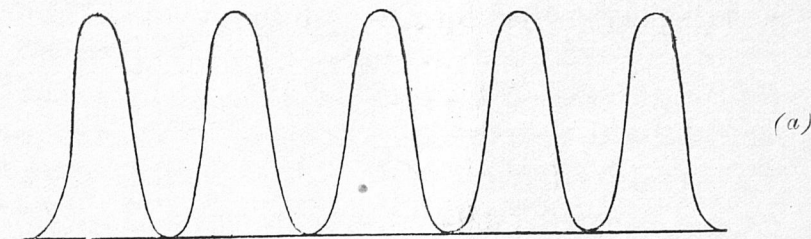
The intensity curve has been plotted out in Plate I (a) for the case $a=1.5$ cm. and $d=0.2$ cm. It will be seen that in this plane $I_{\max.}$ begins nearly with a value $(1+1)^2=4$, and drops down gradually in successive fringes to a limiting value of $\left(1 + \frac{1}{\sqrt{3}}\right)^2 = 2.49$.

$I_{\min.}$ increases from a value nearly zero to a limiting value $\left(1 - \frac{1}{\sqrt{3}}\right)^2 = 0.18$. The 'visibility' of the successive fringes in the plane of observation, therefore, decreases slowly.

At the plane $y=0$, the illumination is given by

$$I = 1 + \frac{1}{3} - \frac{2}{\sqrt{3}} \cos\phi \quad \dots (5)$$

The intensity curve is shown in Plate I (b). $I_{\max.}$



Illumination Curves.

has a constant value of $\left(1 + \frac{1}{\sqrt{3}}\right)^2$, and $I_{\min}$, a value $\left(1 - \frac{1}{\sqrt{3}}\right)^2$. The 'visibility' of the fringes is thus stationary along this plane for a considerable distance from the edge.

Passing on to consider the distribution of Intensity in any plane $Q'P'$ below the plane $y=0$, we have, as remarked above, to add to the effect of the direct and the reflected rays a diffraction effect. We shall represent the latter effect by that due to a single source placed at the edge O , the amplitude of the disturbance due to it, at a point in the region bounded by OX and OY being given by Sommerfeld's expression*

$$\frac{1}{4\pi} \sqrt{\frac{\lambda}{r}} \cos \left(kr - nt + \frac{\pi}{4} \right) \left\{ \pm \frac{1}{\cos \frac{\varphi + \varphi'}{2}} - \frac{1}{\cos \frac{\varphi - \varphi'}{2}} \right\} \quad (6)$$

where r is the distance of the point from the edge O , φ, φ' are the angles which the diffracted and the incident beams respectively make with the direction XO .

If we denote the angle $P'OY$ by α

$$\varphi = 3\pi/2 - \alpha \quad \varphi' = \pi/2.$$

so that expression (6) becomes

$$\frac{1}{4\pi} \sqrt{\frac{\lambda}{r}} \cos \left(kr - nt + \frac{\pi}{4} \right) \left(\pm 1 - \frac{1}{\sin \frac{\alpha}{2}} \right)$$

$$= -\frac{1}{4\pi} \frac{2}{\alpha} \sqrt{\frac{\lambda}{r}} \cos \left(kr - nt + \frac{\pi}{4} \right)$$

since α is small so far as our investigation is concerned.

Now $r = P'O = y + \frac{x^2}{2y}$; $\alpha = \frac{x}{y}$, approximately,

* Sommerfeld—*loc. cit.*

so that the above expression reduces to

$$-\frac{\sqrt{y\lambda}}{2\pi x} \cos \left(k \cdot y + \frac{x^2}{2y} - nt + \frac{\pi}{4} \right) \quad (\text{approx.}).$$

The total disturbance at P' is thus

$$\xi = \cos(ky - nt) - \sqrt{s} \cos(ky - nt + k \cdot \frac{2a\theta^2 + 2y\theta^2}{2y}) - \frac{\sqrt{y\lambda}}{2\pi x} \cos \left(k \cdot y + \frac{x^2}{2y} - nt + \frac{\pi}{4} \right).$$

Remembering that, since $x = 3a\theta^2/2 + 2y\theta$, $\frac{x^2}{2y} = 2y\theta^2 + 3a\theta^4$ we get for the intensity of illumination at any point, the expression $I = 1 + S - 2\sqrt{S} \cos \chi$... (7)

$$\text{where } S = s + \frac{y\lambda}{4\pi^2 x^2} + \frac{\sqrt{sy\lambda}}{\pi x} \cos \left(ka\theta^3 + \frac{\pi}{4} \right) \quad \dots (8)$$

$$\chi = k(2a\theta^3 + 2y\theta^2) + \epsilon \quad \dots (9)$$

$$\text{and } \tan \epsilon = \frac{\frac{\sqrt{y\lambda}}{\pi x} \sin \left(ka\theta^3 + \frac{\pi}{4} \right)}{2\sqrt{s} + \frac{\sqrt{y\lambda}}{\pi x} \cos \left(ka\theta^3 + \frac{\pi}{4} \right)} \quad \dots (10)$$

Equations (4) and (7) give for the positions of the maxima and minima of illumination

$$\left. \begin{aligned} 3a\theta^2/2 + 2y\theta &= x \\ 2a\theta^3 + 2y\theta^2 &= m\lambda/2 - \frac{\epsilon\lambda}{2\pi} \end{aligned} \right\} \quad \dots (11)$$

We see that the introduction of the diffraction term has slightly changed the positions of the maxima and minima. The magnitude of the change depends upon ϵ , which is zero when $y=0$ (equation 10), and steadily increases with y to a limiting value of $\left(ka\theta^3 + \frac{\pi}{4} \right)$ or $\frac{\pi}{4}$ since $ka\theta^3$ is negligible over the first few bands when y is sufficiently large. By actual calculation it is found that this limit is practically reached, when y is over three times the radius of the cylinder. Under these conditions, equations (11) reduce to

$$x^2 = y\lambda(4m-1)/4. \quad \dots (12)$$

Formula (12) is identical with Schuster's formula for the case of diffraction of plane waves by a straight edge.

Returning to equation (7), we see that the intensity of illumination of the successive maxima and minima is given by

$$\left. \begin{aligned} I_{\max.} &= (1 + \sqrt{S})^2 \\ I_{\min.} &= (1 - \sqrt{S})^2 \end{aligned} \right\} \quad \dots (13)$$

where S is given by equation (8).

The intensity curve for a plane 5 mm. behind the "edge" is shown in Plate I (c). It will be seen that the ratio of the minima to the maxima is considerably greater than in (a) and (b). Calculation also shows that the visibility of the successive fringes in this plane of observation should decrease, though somewhat slowly. For still greater distances from the edge of the cylinder, the illumination curves become practically identical with that of the Fresnel type due to a straight edge, the intensity of the reflected rays becoming negligible in comparison with that of the incident and the diffracted rays. The ordinates of the curves (a), (b), and (c), (though not the abscissæ) have all been drawn to the same scale, and the curves illustrate the fact that the luminosity of the field as a whole decreases as we recede from the cylinder.

§ 3. Photometric Study of the field.

The formulæ obtained above have been tested by two independent methods—(1) by photometric comparison of the maxima and minima of illumination, and (2) by determination of their relative positions.

A small polished cylinder of glass, of about 1.5 cm. radius, was used. It was mounted on one of the

stands of an optical bench, and a microscope objective mounted on another of those stands was brought up close to the cylinder. Light from a narrow slit was passed through a collimating lens and was allowed to fall grazingly on the cylinder. The field was viewed through a micrometer eye-piece, placed at a distance behind the objective. By moving the cylinder towards or away from the objective, the phenomena at different planes $y=d$ could be observed.

The photometric arrangement used to study the relative intensity of the fringes was based on a Polarization method. The beam of light was plane polarized by passage through a Nicol before falling on the cylinder. The eye-piece, (a low-power one) was moved off to a pretty large distance behind the objective, and two narrow slits cut out of aluminium foil and pasted on two glass strips, were mounted, one above the other, between the eye-piece and the objective, so as to allow a small relative motion which could be controlled by a micrometer-screw. A thin mica plate was also fixed up on the upper one (which was movable in the actual experiment), the thickness of the plate and its orientation with respect to the slit being adjusted by trial so that, under the conditions of the experiment, it circularly polarized the light falling on it. The field was viewed through another Nicol fitted with a graduated circle and mounted just behind the slits.

The lower slit was always set on the first bright band, while the upper one was set successively on the different maxima and minima. Equality of illumination of the upper and lower slits was obtained in each case, by rotating the analysing Nicol. The reading for the

crossed position of the analyser being also taken, the ratio of the intensities of illumination could be easily calculated. Thus if ψ be the orientation of the analyser in any case (ψ being reckoned from the crossed position) and I, I_0 the intensities of illumination of the upper and lower slits respectively,

$$I/I_0 = \sin^2 \psi.$$

Readings were taken for the first few bands on the planes (1) $y=0$, (2) $y=0.5$ cm., and are shown in Table I with the corresponding values calculated from theory.

TABLE I.

n	y=0.				y=0.5 cm.			
	MAXIMUM.		MINIMUM.		MAXIMUM.		MINIMUM.	
	I _n /I ₀ .		I _n /I ₀ .		I _n /I ₀ .		I _n /I ₀ .	
	Obsd.	Calcd.	Obsd.	Calcd.	Obsd.	Calcd.	Obsd.	Calcd.
1	1.00	1.00	0.09	0.07	1.00	1.00	0.46	0.42
2	0.96	1.00	0.08	0.07	0.91	0.93	0.45	0.44
3	1.00	1.00	0.12	0.07	0.91	0.91	0.52	0.46
4	0.94	1.00	0.15	0.07	0.83	0.90	0.52	0.46

The discrepancies are within the limits of experimental error, so that the theory developed above appears to be substantially correct.

As has been remarked already, the theory was also tested by measurements of the positions of the minima of illumination in different parts of the field. Some explanation is here necessary with regard to the measurements of fringes in a plane in advance of the "edge" (y negative). With the ordinary arrangement

as described above, if the microscope is moved near, so that its focal plane may be in advance of the edge, we are unable to see the exact phenomena in that plane, since the light has to come past the "edge" before it can fall on the objective; and secondly, as has been fully described in Mr. Basu's* paper, the field is complicated by the occurrence of the caustic and its accompanying fringes, formed by reflection from the surface of the cylinder. This difficulty was got over in the present work by turning the cylinder around its axis, till the desired plane of observation coincided with the boundary of the polished surface. What is meant may be better understood by a reference to fig. 1, the process described being equivalent to cutting off the cylinder along CQ and removing the lower half. It is not easy, however, with this arrangement, directly to determine the value of y when it is negative; and in Table II below it has been obtained by calculation from a pair of readings. The source of light was a quartz mercury lamp with a green ray filter.

TABLE II.

Fringes between the Cylindrical Edge and the Source of light.

n	$y=0$		$y=-2.0$ mm.		$y=-2.9$ mm.	
	$x_n - x_1$		$x_n - x_1$		$x_n - x_1$	
	Obsd.	Calcd.	Obsd.	Calcd.	Obsd.	Calcd.
3	0.0169	0.0169	0.0082	0.0082	0.0062	0.0060
5	0.0307	0.0302	0.0165	0.0163	0.0123	0.0120
7	0.0420	0.0417	0.0240	0.0240	0.0183	0.0182
9	0.0524	0.0522	0.0310	0.0317	0.0235	0.0235
11	0.0620	0.0619	0.0378	0.0395	0.0290	0.0293
			mms.			

* Basu—*loc. cit.*TABLE III.
Fringes behind the Cylindrical Edge.

n	$d=0.1$ cm.		$d=0.3$ cm.		$d=0.5$ cm.		$d=0.7$ cm.	
	$x_n - x_1$		$x_n - x_1$		$x_n - x_1$		$x_n - x_1$	
	Obsd.	Calcd.	Obsd.	Calcd.	Obsd.	Calcd.	Obsd.	Calcd.
2	0.0159	0.0158	0.0250	0.0251	0.0341	0.0326	0.0370	0.0370
3	0.0284	0.0284	0.0439	0.0450	0.0580	0.0580	0.0660	0.0661
4	0.0385	0.0390	0.0600	0.0609	0.0776	0.0782	0.0887	0.0897
5	0.0483	0.0480	0.0740	0.0750	0.0928	0.0938		
6	0.0568	0.0570	0.0867	0.0879	0.1116	0.1134		
7	0.0651	0.0650	0.0990	0.0997				
8	0.0715	0.0723	0.1108	0.1110				
9	0.0794	0.0796						
10	0.0862	0.0863						

 $\lambda = 5461$ A. U.; $a = 1.54$ cm.

The agreement between the calculated and the observed values is very close and confirms the theory.

§ 4. *The Loci of maxima and minima of Illumination.*

These curves have an interesting property which may be briefly considered here. The equation to the loci is obviously

$$r(1 - \cos 2\theta) = n\lambda/2$$

or $r\theta^2 = n\lambda/4$ approx. ... (14)

where r is the distance of the point from the surface of the cylinder measured along the reflected ray which passes through the point. The shape of the curves is indicated in Plate II (thick lines), for the case when $a = 5$ inches, $\lambda = 0.16$ inch, λ being taken so large for convenience of representation to scale; only the 1st, 3rd, 5th &c., loci are shown.

The equation to the loci can also be got in terms of θ , and x or y . Thus on eliminating y from equations (3) and (4) we get

$$x = \frac{m\lambda}{2\theta} - \frac{a\theta^2}{2} \quad \dots \quad (15)$$

which gives the abscissæ of the points at which the loci cut the straight lines $\theta = \text{const.}$

The ordinates of these points are given by

$$2y\theta = x - \frac{3}{2}a\theta^2 = \frac{m\lambda}{2\theta} - 2a\theta^2$$

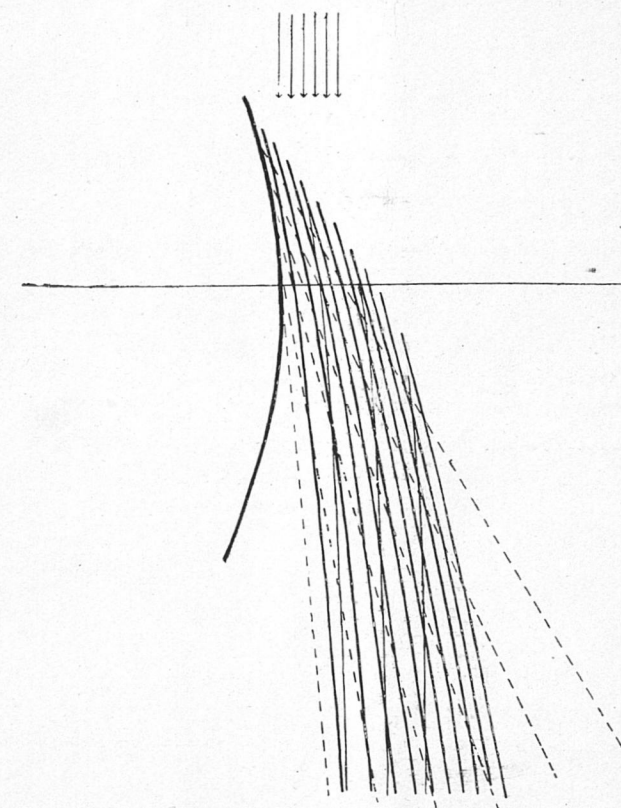
$$\therefore y = \frac{m\lambda}{4\theta^2} - a\theta. \quad \dots \quad (16)$$

From (15) and (16) it is seen that

$$dx = -\left(\frac{m\lambda}{2\theta^2} + a\theta\right) \delta\theta,$$

and $dy = -\left(a + \frac{m\lambda}{2\theta^3}\right) \delta\theta$

so that $\frac{dy}{dx} = \theta$



Heavy lines represent the loci of minimum illumination,
thin lines the "mean lines of flow" of energy,
and dotted lines the reflected rays.

At any point, therefore, the curves bisect the angle between the directions of the direct and the reflected rays which pass through that point.

As might be expected, the formulæ obtained above for the case of diffraction by a cylinder, reduce to the ordinary formulæ for diffraction by a straight edge, on writing $\alpha=0$, provided, of course, that the light from the other side of the cylinder is cut off by a semi-infinite plane extending to the left of the origin O.

For then, equations (11) become

$$\left. \begin{aligned} x &= 2y\theta \\ m\lambda/2 - \epsilon\lambda/2\pi &= 2y\theta^2 \end{aligned} \right\}$$

and $\epsilon = \frac{\pi}{4}$, so that $x^2 = y\lambda (4m-1)/4$.

which is Schuster's formula for diffraction of plane waves at straight edge. The results regarding the loci of maxima and minima will also apply for diffraction at a straight edge, under the same conditions.

§ 5. On the flow of energy in the field.

We will first take into account only the effects due to the interference of the direct and the reflected rays. The effect due to diffraction at the edge of the cylinder can be brought in later as a correction.

Let us assume, for simplicity, that the light is polarized in the plane of incidence, so that the electric intensity is perpendicular to that plane, and the magnetic intensity lies in it. Then at any point P (fig. 1.) the resultant electric intensity is

$$E = \left[\cos n \left(t - \frac{r_1}{c} \right) + \left(\frac{\rho}{\rho + r_2} \right)^{\frac{1}{2}} \cos \left\{ n \left(t - \frac{r_2}{c} \right) + \pi \right\} \right]$$

where $r_1 = PM$, $r_2 = PQ$ and ρ is the radius of curvature

of the reflected wave at Q. The expression $\left(\frac{\rho}{\rho+r_2}\right)^{\frac{1}{2}}$ will in the rest of the paper be denoted by K. The resultant magnetic intensity at P is

$$\vec{H} = \cos n \left(t - \frac{r_1}{c} \right) + K \cos \left\{ n \left(t - \frac{r_2}{c} \right) + \pi \right\}$$

Let now $\mathbf{k}_1, \mathbf{k}_2$ be unit vectors at P in the direction of the direct and the reflected rays respectively. The flow of energy is determined by the Poynting vector S where

$$\begin{aligned} \vec{S} &= \frac{c}{4\pi} [\vec{E} \vec{H}] = \frac{c}{4\pi} \left\{ \cos n \left(t - \frac{r_1}{c} \right) + K \cos \left\{ n \left(t - \frac{r_2}{c} \right) + \pi \right\} \right\} \\ &\quad \times \left\{ \mathbf{k}_1 \left[\cos n \left(t - \frac{r_1}{c} \right) \right] + \mathbf{k}_2 \left[K \cos n \left(t - \frac{r_2}{c} \right) + \pi \right] \right\}, \\ \text{or } \frac{4\pi \vec{S}}{c} &= \mathbf{k}_1 \left[\cos^2 \chi + K \cos \chi \cos \chi' \right] + \mathbf{k}_2 \left[K \cos \chi \cos \chi' + K^2 \cos^2 \chi' \right] \end{aligned}$$

$$\text{where } \chi = n \left(t - \frac{r_1}{c} \right) \text{ and } \chi' = n \left(t - \frac{r_2}{c} \right) + \pi.$$

The time-mean $\bar{S}$ of the flow of energy is given by

$$\begin{aligned} \frac{4\pi \bar{S}}{c} &= \frac{1}{2} \mathbf{k}_1 \left[1 + K \cos (\chi' - \chi) \right] + \frac{1}{2} \mathbf{k}_2 \left[K^2 + K \cos (\chi' - \chi) \right] \\ \frac{8\pi \bar{S}}{c} &= \mathbf{k}_1 [a_1] + \mathbf{k}_2 [a_2] \end{aligned}$$

$$\text{if } a_1 = 1 + K \cos (\chi - \chi') \text{ and } a_2 = K^2 + K \cos (\chi' - \chi)$$

If φ be the angle which the direction of $\bar{S}$ makes with that of the incident rays, it can be easily shown that

$$\tan \varphi = \frac{a_2 \sin 2\theta}{a_1 + a_2 \cos 2\theta} = \frac{2a_2 \theta}{a_1 + a_2}$$

since θ is small.

$$\text{Thus } \tan \varphi = \frac{2\theta \{ K^2 + K \cos (\chi' - \chi) \}}{1 + 2K \cos (\chi' - \chi) + K^2} \dots \dots (17)$$

$$\text{now } \chi' - \chi = \frac{n(r_2 - r_1)}{c} + \pi = \frac{n}{c} 2r_2 \theta^2 + \pi.$$

Hence equation (17) becomes

$$\tan \varphi = \frac{2\theta \{ K^2 - K \cos \frac{n}{c} 2r_2 \theta^2 \}}{1 + K^2 - 2K \cos \frac{n}{c} 2r_2 \theta^2} \dots \dots (18)$$

The current of energy at the point is

$$\begin{aligned} \bar{S} &= \frac{c}{8\pi} \left\{ a_1^2 + a_2^2 + 2a_1 a_2 \cos 2\theta \right\}^{\frac{1}{2}} \\ &= -\frac{c}{8\pi} \left(1 + K^2 - 2K \cos \frac{n}{c} 2r_2 \theta^2 \right) \dots \text{(approx.)} \dots (19) \end{aligned}$$

The lines of flow of energy are determined by the condition that at any point (r_2, θ) the inclination to the

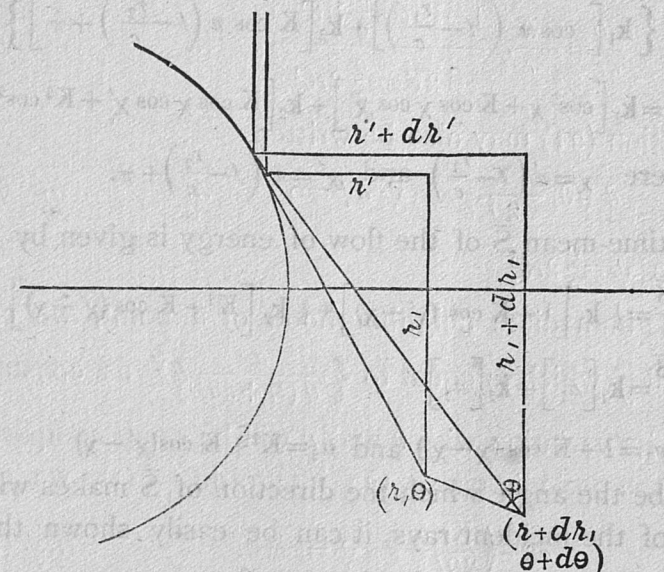


Fig. 2

direct rays is φ , where φ is given by equation (18). From this, we can get the differential equation to the lines of flow in terms of r_2 and θ , or dropping the suffix, in terms of r and θ . It is seen from fig. 2, that

$$\tan \varphi = \frac{dr' - a\theta d\theta}{dr_1 - a d\theta}$$

But $r' = r \sin 2\theta = 2r\theta$; $r_1 = r \cos 2\theta = r$ (approx.)

$$\therefore \tan \varphi = \frac{2(r d\theta + \theta dr) - a\theta d\theta}{dr - a d\theta}$$

$$\text{or } \tan \varphi \left(1 - a \frac{d\theta}{dr}\right) = 2\theta + (2r - a\theta) \frac{d\theta}{dr} \dots \quad (20)$$

From (18) and (20), we get

$$\frac{d\theta}{dr} (2r - a\theta) + 2\theta = \left(1 - a \frac{d\theta}{dr}\right) \frac{2K\theta \left(K - \cos \frac{n}{c} 2r\theta^2\right)}{1 + K^2 - 2K \cos \frac{n}{c} 2r\theta^2}$$

which on reduction becomes

$$\frac{d\theta}{dr} = -\frac{\theta}{r} \cdot \frac{1 - K \cos \frac{n}{c} 2r\theta^2}{1 - 2K \cos \frac{n}{c} 2r\theta^2} \dots \quad (21)$$

Equation (21) may also be written

$$d(r\theta) - \frac{K}{2\theta} \cdot \frac{c}{n} \cos \psi d\psi = 0 \dots \quad (22)$$

where $\psi = \frac{n}{c} 2r\theta^2$.

In the immediate neighbourhood of a point, we can regard $\frac{K}{\theta}$ which is equal to $\left\{\frac{a}{\theta(2r+a\theta)}\right\}^{\frac{1}{2}}$ as constant since its variation with θ and r will be small compared with the periodic part. On integrating (22), we get

$$r\theta - \left(\frac{K}{2\theta}\right) \frac{c}{n} \sin \psi = \text{const.} \dots \quad (23)$$

which determines the shape of the lines of flow in the immediate neighbourhood of a point. The points of intersection of these curves with the loci of maxima and minima of illumination ($\psi = m\pi$), are given by the equation

$$r\theta = \text{const.} \dots \quad (24)$$

This gives the "mean lines of flow" about which energy "crinkles" down. They are shown in Plate II (thin lines). We find that so long as θ is not very large, these curves are inclined to the direct rays at angles smaller than those corresponding to the maxima and minima loci. If we imagine the latter set of curves as forming successive bright and dark tubes, energy will flow down across the tubes, its direction being periodically shifted such that it tends to flow along the bright tubes and to cut across dark tubes. The shift in its direction goes through one complete cycle, as the energy passes from one dark tube, to the next dark tube, or from one bright tube to the next, so that the 'Wave-length' of a crinkle in the neighbourhood of a point (r, θ) may be determined by finding the distance, along the curve $r\theta = \text{const.}$, between two successive points of intersection of that curve and the family of curves $r\theta^2 = \frac{m\lambda}{2}$ (the loci of minima of illumination).

Thus

$$r\theta^2 = \frac{m\lambda}{2}; r\theta = C$$

$$\text{giving } \theta = \frac{m\lambda}{2C} \text{ or } \delta\theta = \frac{\lambda}{2C}$$

if $\delta\theta$ be the difference in the θ -co-ordinates between two successive points of intersection. Also if σ be the arc measured along the curve $r\theta = C$, we have from equation (20)

$$\left(\frac{d\sigma}{d\theta}\right)^2 = \left\{(2r - a\theta) + 2\theta \frac{dr}{d\theta}\right\}^2 + \left\{\frac{dr}{d\theta} - a\right\}^2 \dots \quad (25)$$

Along the curve $r\theta = C$, $\frac{dr}{d\theta} = -\frac{r}{\theta}$.

Hence from (25) $\left(\frac{d\sigma}{d\theta}\right)^2 = a^2 + \frac{r^2}{\theta^2} + \frac{2ar}{\theta}$ approximately.

$$\frac{d\sigma}{d\theta} = a + \frac{r}{\theta} = a + \frac{C}{\theta^2}.$$

If l be the "wave-length" of a crinkle,

$$l = \left(a + \frac{C}{\theta^2} \right) \delta\theta = \left(a + \frac{C}{\theta^2} \right) \frac{\lambda}{2C} \quad \text{approx.}$$

$$= \frac{a\lambda}{2C} + \frac{\lambda}{2\theta^2} \quad \dots \quad \dots \quad \dots \quad (26)$$

C being the same, l increases as θ decreases, the rate of increase getting larger as the absolute value of θ diminishes; $l = \infty$ when $\theta = 0$. Again as we move away from the cylinder to the right, both C and θ increase, so that l decreases. These points are brought out in Plate III.

Turning back to equation (23), the lines of flow near the point $r_1\theta_1$ are given by

$$r\theta = C - \frac{K_1}{\theta_1} \frac{\lambda}{4\pi} \sin \psi.$$

The curves will obviously lie between the curves

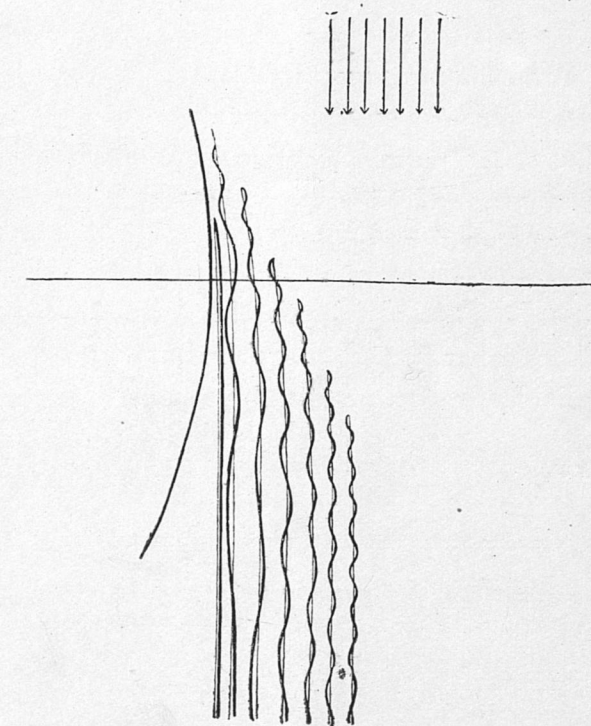
$$r\theta = C \pm \frac{K_1}{\theta_1} \frac{\lambda}{4\pi}.$$

The deviation from the mean line $r\theta = C$ is proportional to

$$\frac{K_1}{\theta_1} = \left\{ \frac{a}{\theta (2r + a\theta)} \right\}^{\frac{1}{2}}$$

As either r or θ increases, this will decrease. The "amplitude" of these crinkles therefore gets smaller and smaller as r and θ increase, and the crinkles vanish at sufficiently large distances from the cylinder.

The shape of the lines of flow very near the surface of the cylinder is of special interest. The energy which comes crinkling down successive loci of maxima and minima of illumination, when it reaches the first maximum, flows down in a smooth curve which will



Lines of flow of energy in different parts of the field.

meet the mean line of flow only at infinity. The energy does not crinkle along after it has crossed the first maximum of illumination.

It is thus seen that the introduction of a perfectly reflecting cylinder into a field through which plane light waves are passing, has the effect of (1) altering the general direction of flow of energy, and (2) giving a crinkled microscopic structure to the energy current at any point. But we have still to seek an explanation as to how a flow of energy in the manner described above leads to the actual distribution of maxima and minima of illumination in the field. The current of energy at any point is by (19)

$$\bar{S} = \frac{c}{8\pi} \left(1 - 2K \cos \frac{n}{c} 2r\theta^2 + K^2 \right)$$

and this varies from point to point along each line of flow, being maximum and minimum respectively where it cuts successive curves $2r\theta^2 = m\lambda/2$. This variation of the current of energy along its own line of flow can be explained only as due to the change in cross-section of the tube of flow formed by two lines of flow close to each other. The conception of energy as flowing through tubes must therefore give us a better idea of what happens in the field.

The curves which are at every point normal to the lines of flow, *i.e.*, the curves analogous to equipotential curves, can easily be obtained. For these curves,

$$\tan \phi' = - \frac{1 - 2K \cos \frac{n}{c} 2r\theta^2 + K^2}{2\theta \left(K^2 - K \cos \frac{n}{c} 2r\theta^2 \right)} \quad [\text{cp. eqn. (2)}]$$

The differential equation in terms of θ and r is found to be

$$(dr - a d\theta) + \frac{2K}{1+K^2} \cos \frac{n}{c} 2r\theta^2 \left[(a - 2r\theta) d\theta - dr \right] = 0.$$

In the immediate neighbourhood of a point, we can as before leave out all variations other than the periodic one, and integrate. Then we get

$$r - a\theta + \frac{2K_1}{1+K_1^2} \sin \frac{n}{c} 2r\theta^2 \left[\frac{a - 2r_1\theta_1}{4r\theta^{\frac{n}{c}}} - \frac{1}{2\theta^2 \frac{n}{c}} \right] = \text{const.} \quad (27)$$

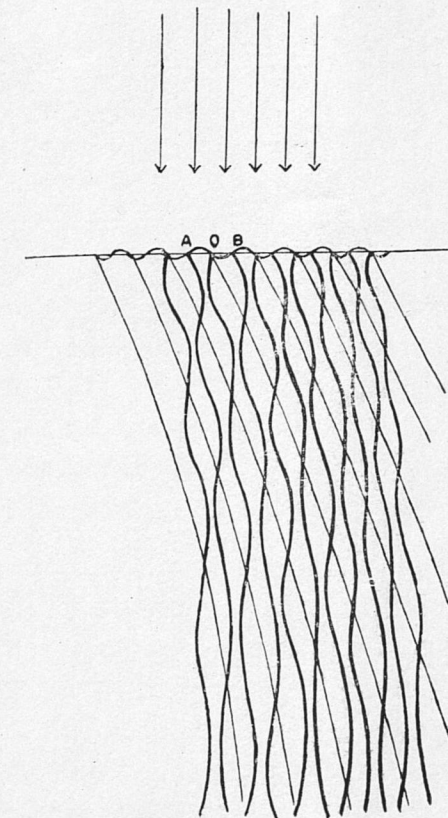
which cuts the successive loci of maxima and minima of illumination at points lying on the curve

$$r - a\theta = \text{const.} \quad \dots \quad \dots \quad (28)$$

which is therefore the mean curve about which the actual curves crinkle round. In the rectangular co-ordinates (x, y) , equation (28) becomes

$$y = \text{const.}$$

i.e., the mean curves are straight lines parallel to the x -axis. The curves are shown in Plate IV, in relation to the lines of flow and to the loci of minimum illumination. Suppose one of these curves cuts two successive minima loci at points A and B. Then by equation (27), there is one complete crinkle between A and B. Consider the tube of flow bounded by the lines of flow which pass through A and B. Since the flow of energy should be everywhere normal to the wavy curve AOB, energy is concentrated in the right half of the tube and 'rarefied' in the left half. If we draw the line of flow passing through a point O, midway between A and B, we see that we can conceive of the tube AB as made up of two tubes each of which widens and contracts periodically, and one of which is so shifted relatively to the other, that the broadened part of one falls by



Microscopic structure of the field.

the side of the narrow part of the other. We see also from Plate IV, that the contracted parts of successive tubes lie along the loci of max. illumination.

We may now take into account the diffraction at the edge of the cylinder and obtain the correction to our results necessary for points below the plane $y=0$. Using Sommerfeld's expression as we have done before, the electric intensity at a point P' (fig. 1.) due to this alone is from (6)

$$\begin{aligned} & \frac{1}{4\pi} \sqrt{\frac{\lambda}{r'}} \cos \left\{ \frac{n}{c} (r' - ct) + \frac{\pi}{4} \right\} \left\{ 1 - \frac{1}{\sin \frac{\alpha}{2}} \right\} \\ &= -\frac{1}{2\pi\alpha} \sqrt{\frac{\lambda}{r'}} \cos \left\{ \frac{n}{c} (r' - ct) + \frac{\pi}{4} \right\} \end{aligned}$$

since α is small; $r' = P'O$.

Let $P'Q = r$, and $P'M' = r_1$.

Then $r' = (r_1 - a\theta) \sec \alpha = (r_1 - a\theta) \sec 2\theta$
 $= -a\theta + r_1 (1 + 2\theta^2)$ approximately.

$$\therefore r' + a\theta = r_1 (1 + 2\theta^2).$$

$$\alpha = \frac{2r\theta}{r - a\theta}.$$

If E and H be the resultant electric and magnetic intensities at P',

$$\begin{aligned} E = & \cos n \left(t - \frac{r_1}{c} \right) - K \cos n \left(t - \frac{r'}{c} \right) \\ & - \frac{1}{2\pi\alpha} \sqrt{\frac{\lambda}{r'}} \cos \left\{ n \left(t - \frac{r' + a\theta}{c} \right) - \frac{\pi}{4} \right\} \end{aligned}$$

$$\text{and } H = \cos n \left(t - \frac{r_1}{c} \right) - K \cos n \left(t - \frac{r'}{c} \right)$$

$$- \frac{1}{2\pi\alpha} \sqrt{\frac{\lambda}{r'}} \cos \left\{ n \left(t - \frac{r' + a\theta}{c} \right) - \frac{\pi}{4} \right\}$$

If k_1, k_2, k_3 , be unit vectors measured along r_1, r and r' respectively, the time-mean of the Flow of energy is found to be given by

$$\begin{aligned} \frac{8\pi\bar{S}}{c} = & k_1 \left[1 - K \cos \frac{n}{c} 2r\theta^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \right] \\ & + k_2 \left[K^2 - K \cos \frac{n}{c} 2r\theta^2 + KK' \cos \frac{\pi}{4} \right] \\ & + k_3 \left[K'^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) + KK' \cos \frac{\pi}{4} \right] \end{aligned}$$

K' being written for $\frac{1}{2\pi\alpha} \sqrt{\frac{\lambda}{r}}$.

Resolving the vectors along and perpendicular to the direction of the direct rays, we get

$$\left. \begin{aligned} \frac{8\pi}{c} \bar{S}_x = & 1 + K^2 + K'^2 - 2K \cos \frac{n}{c} 2r\theta^2 - 2K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \\ & + 2KK' \cos \frac{\pi}{4} \\ \frac{8\pi}{c} \bar{S}_y = & 2\theta \left\{ -K \cos \frac{n}{c} 2r\theta^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \right. \\ & \left. + 2KK' \cos \frac{\pi}{4} + K^2 + K'^2 \right\} \end{aligned} \right\} 29$$

where θ^2 is neglected in comparison with unity and α and θ are considered to be the same in the small term $\bar{S}_y$. Since K, K' are each small in the present problem, terms involving their products and squares can be neglected. The expression for $\tan \varphi$ may then be written

$$\tan \varphi = \frac{\bar{S}_y}{\bar{S}_x} = \frac{2\theta \left\{ -K \cos \frac{n}{c} 2r\theta^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \right\}}{1 - 2K \cos \frac{n}{c} 2r\theta^2 - 2K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right)} \quad (30)$$

From equation (20)

$$\frac{d\theta}{dr} (2r - a\theta) + 2\theta = \left(1 - a \frac{d\theta}{dr} \right) \tan \varphi$$

and (30) becomes on reduction

$$\frac{d\theta}{dr} = -\frac{\theta}{r} \cdot \frac{1 - K \cos \frac{n}{c} 2r\theta^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right)}{1 - 2K \cos \frac{n}{c} 2r\theta^2 - 2K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right)} \dots (31)$$

$$\begin{aligned} \text{Now } K \cos \frac{n}{c} 2r\theta^2 + K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \\ = D \cos \left(\frac{n}{c} 2r\theta^2 + \epsilon \right) \end{aligned}$$

$$\text{if } D^2 = (K + K'/\sqrt{2})^2 + \frac{K'^2}{2} = K^2 + K'^2 + KK' \cdot \sqrt{2}$$

$$\text{and } \tan \epsilon = \frac{K'}{K + K'\sqrt{2}}.$$

Then (30) may be written

$$\frac{d\theta}{dr} = -\frac{\theta}{r} \frac{1 - D \cos \left(\frac{n}{c} 2r\theta^2 + \epsilon \right)}{1 - 2D \cos \left(\frac{n}{c} 2r\theta^2 + \epsilon \right)} \dots (32)$$

In the neighbourhood of any point, D and ϵ can be regarded as constant and equation (32) integrated.

$$r\theta = C - \frac{D_1}{\theta_1} \sin \left(\frac{n}{c} 2r\theta^2 + \epsilon \right)$$

The "mean lines of flow" of energy are still given by $r\theta = C$.

but the actual lines of flow which wind about them cut them along the curves

$$\frac{n}{c} 2r\theta^2 + \epsilon = m\pi \quad \dots \dots (33)$$

instead of the curves $\frac{n}{c} 2r\theta^2 = m\pi$. It may be noted that

it is equation (33) that determines the position of the maxima and minima of illumination when the effect of diffraction is also taken into account, so that the points of intersection of the lines of flow with the loci of points of max. and min. illumination still lie on the "mean lines of flow."

If the squares and products of K , are not neglected, it can be shown that the mean lines are given by the equation

$$r d\theta (1 + K'^2 + \sqrt{2} KK') + \theta dr = 0.$$

$$\text{or } r\theta^{1+\alpha} = \text{const.}$$

$$\text{where } \alpha = K'^2 + KK' \cdot \sqrt{2}$$

It will be interesting finally to deduce the results for the case of diffraction at a straight edge from the above investigation. Putting $\alpha = 0$, K becomes zero also. We have then directly from equations (29)

$$\tan \varphi = \frac{2\theta \left\{ K'^2 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right) \right\}}{1 + K'^2 - 2K' \cos \left\{ \frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right\}}$$

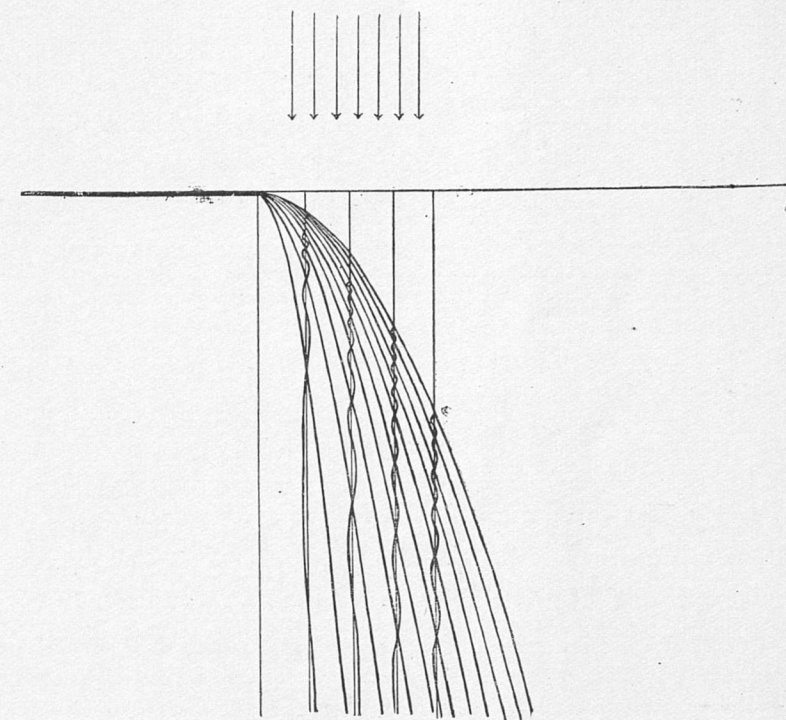
$$\text{and } \frac{d\theta}{dr} = -\frac{\theta}{r} \cdot \frac{1 - K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right)}{1 - 2K' \cos \left(\frac{n}{c} 2r\theta^2 + \frac{\pi}{4} \right)}$$

an expression very similar to (21).

The "mean lines of flow" are given by $r\theta = \text{const.}$ or in the rectangular co-ordinates used, by $x = \text{const.}$ They are straight lines parallel to the Y-axis, the "mean direction" of energy flow being apparently not altered by the presence of the edge. The actual lines of flow cut the successive loci of points of max. and min. illumination (*i. e.*, the curves $2r\theta^2 + \frac{\lambda}{8} = m \frac{\lambda}{2}$) at points which lie along the mean lines of flow. The shape of these lines of flow in the diffraction field due to a straight edge is indicated in the figure in Plate V, which has been drawn for $\lambda = 0.5$ inch for convenience of representation.

Chinmayanandam.

PLATE V.



Lines of energy flow in the field due to diffraction at a straight edge.

§ 6. Summary and Conclusion.

The present paper deals with the distribution and flow of energy in the immediate neighbourhood of a perfectly reflecting cylinder on which plane light waves are grazingly incident in a direction at right angles to its axis. The following are the principal results which are indicated by theory, and have been verified by photometric study of the field.

(a) The positions of the maxima and minima of illumination are determined by eliminating θ from the pair of equations,

$$2y\theta + 3a\theta^2/2 = x; \quad 2\theta^2(y + a\theta) = m\lambda/2$$

or from the pair of equations

$$2y\theta + 3a\theta^2/2 = x; \quad 2\theta^2(y + a\theta) = m\lambda/2 - \epsilon\lambda/2\pi$$

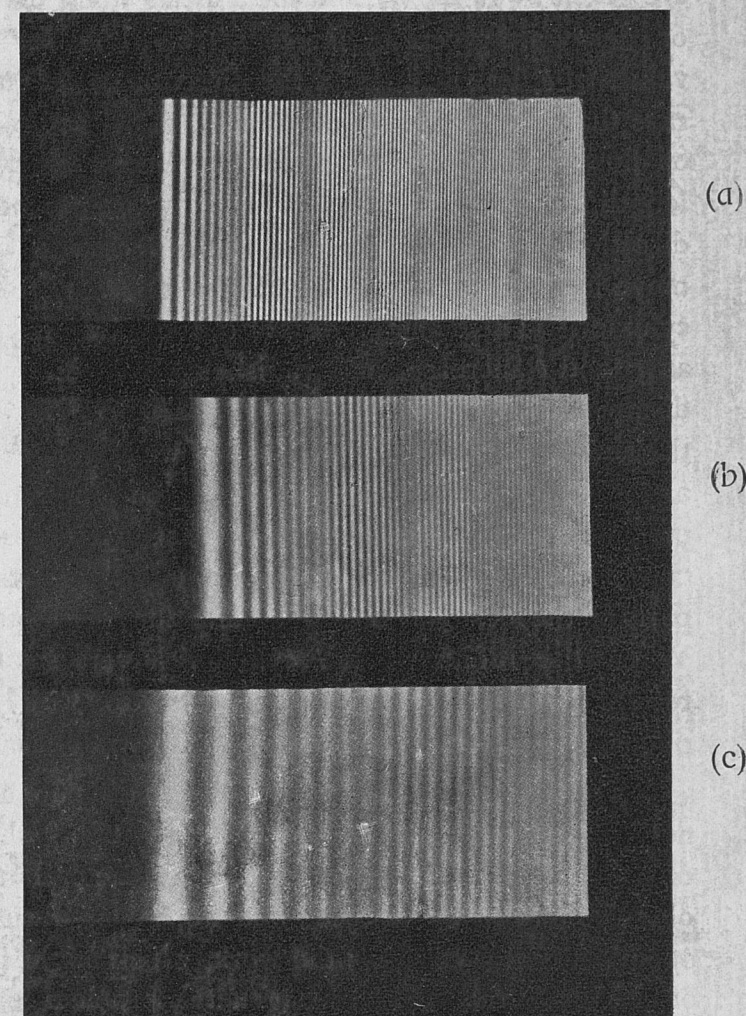
according as the plane of observation is in front of the cylindrical "edge" or behind it. x, y are the co-ordinates of any point, the origin being the "edge" of the cylinder. ϵ is a small angle which for large values of y becomes equal to $\pi/4$.

(b) The visibility of the fringes varies in an interesting manner with the position of the part of the field under observation. It is practically constant over the entire plane of observation when this coincides with the plane passing through the "edge," but falls off when it is moved further away from the source of light, the decrease being greatest for the regions farthest from the surface. When the part of the field under observation is between the "edge" and the source of light, the visibility of the fringes reaches the maximum value at the surface of the cylinder, falling off slowly as we recede from it.

(c) The loci of maxima and minima of illumination are given by $r\theta^2 = m\lambda/4$ and the "mean lines of flow" of energy are given by $r\theta = \text{const.}$ and are for small values of θ , less inclined to the direction of the incident rays than the former set of curves. The actual lines of flow crinkle about these mean lines; the "wavelength" of the crinkles increases as we move along the direction of the incident rays, and decreases as we move in a direction at right angles to it, away from the cylinder. The "amplitude" of the crinkles decreases as r and θ increase, and vanishes at sufficiently large distances from the cylinder. A good conception of the actual phenomena is obtained, if we imagine energy as flowing through tubes which widen and contract periodically, the widened parts of successive tubes lying on the loci of minimum illumination, and the contracted parts on the loci of maximum illumination (see Plate IV). When the radius of the cylinder is very small, the results obtained are practically identical with those obtained in the case of diffraction by a straight edge.

No reference has so far been made to the phenomena noticed within the region of the geometrical shadow of the cylinder. The writer has made some preliminary observations on this subject, and hopes on a suitable opportunity to continue the work, which might prove of interest in relation to the general problem of the diffraction of electro-magnetic waves by cylindrical or spherical surfaces of large radius.

October, 1917.



Illustrating the Interferences in the Field surrounding a Reflecting Cylinder, and the Decrease in the Visibility of the Fringes with Increasing Distances from the Cylinder.

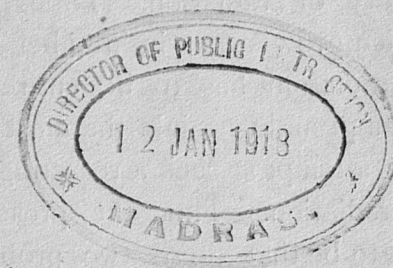
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The Indian Association for the Cultivation of Science.

On Resonance Radiation and the
Quantum Theory.

By T. K. CHINMAYANANDAM, B.A. (HONS.),
*Research Scholar in the Indian Association for the Cultivation
of Science, Calcutta.*

(Reprinted from the Proceedings, Vol. III, Part VI, 1917.)

Calcutta :
1917.



On Resonance Radiation and the Quantum Theory.*

By T. K. CHINMAYANANDAM, B.A. (HONS.)

§ 1. *Introduction.*

The most fascinating problem, and the problem that is engaging the attention of the most eminent scientists of the world at the present day, is the problem of atomic structure. As has been expected for over a long time, the study of the radiation from the atom seems to be the best, and perhaps the only, clue to this problem; and already, as we know, the recent developments in the study of X-rays, and other high frequency radiations, have thrown quite a flood of light into this sub-atomic world. But there seems to be one difficulty, that crops up and mars our progress; if we knew the nature of radiation completely, our way would be easy in solving the problem of atomic structure. But some phenomena have come to light, such as the emission of electrons by X-rays, or ultra-violet rays, which seem to defy all attempts at an explanation on the classical spreading-wave

* Read at the Annual Science Convention, 23rd November, 1917.

theory of radiation. To explain these phenomena, which grow larger in number every day, the theory of radiation has to be entirely recast, and one such attempt and certainly the most important and promising attempt at such a recasting, is the Quantum theory of radiation. Thus the present day position comes out to be that these two problems are closely intertwined, and so are the solutions of those problems; and any promising attempt at the solution of either must be in very close relation with the other.

The Quantum theory of radiation postulates a discontinuity in the absorption and emission of radiations by a substance, and suggests, in fact, that the absorption or emission of a mono-chromatic radiation of frequency ν can take place only in discrete bundles or Quanta as they are called, of energy of amount $h\nu$ where h is Plank's universal constant. In a series of papers in the Philosophical Magazine for 1913, Dr. Bohr has applied this theory to develop a conception of atomic structure, and has shown how the laws of spectral series can be accounted for in the case of Hydrogen, Helium, and also in a general way for other elements. He assumes an atomic model suggested by Sir E. Rutherford, *viz.*, a system with a nucleus of extremely small linear dimensions, and of positive charge Ne (where N is the atomic number of the element and e the electronic charge) and N electrons revolving in concentric rings round the nucleus. Radiation is due to the reformation of a system, which has lost one or more electrons. This binding of the electrons, Bohr assumes, cannot take place in a continuous manner, but in a series of sudden

discontinuous jumps. If the orbits are for simplicity considered to be circular, his main assumption may be regarded as an atomicity of angular momentum, that is to say, the angular momentum of the electron must always be an exact multiple of $\frac{h}{2\pi}$ where h is Plank's constant. The orbits which satisfy this condition are assumed to be non-radiating orbits, and the electron is supposed to emit a homogeneous radiation in passing from one such orbit to another, or from one "Stationary State" to another as they are called. If W_1, W_2 be the energy of the electrons in two such orbits, it can be shown that the amount of energy emitted $= W_2 - W_1$; and this Bohr equates to $h\nu$ where ν is the frequency of the radiation. In a simple case like Hydrogen where there is only one electron, W_1, W_2 can be easily calculated on the principles of ordinary mechanics; Bohr has thus got the formula

$$\nu = K \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\}$$

where n_1, n_2 are integers and $K = \frac{2\pi^2 me^4}{h^3}$. This is of course the Balmer series for Hydrogen, and it may be pointed out that the calculated value of K agrees very closely with the observed value of the Rydberg constant, an agreement on which, in fact, depends much of the success of Bohr's theory. Bohr has not considered, in any great detail, the case of other elements, where the atom has a number of electrons, the case being complicated by the fact that the other electrons will have an effect, besides the nucleus, on the radiating electron. But he has suggested that in the permanent configuration of the atom, or

the configuration of maximum energy, the electrons will all be arranged in a few concentric rings round the nucleus. This is the broad outline of Bohr's theory of spectral series. It may also be pointed out, that Sommerfeld has in a recent paper in the *Annalen Der Physik* for 1916 has carried the theory further and has shown, how by a consideration of the ellipticity of some of the orbits of the radiating electron, we can explain the doublet and triplet structure of the lines in the spectral series of the elements.

It appeared to me a very important and fascinating problem, to try and apply the foregoing ideas, to the phenomena of Resonance Radiation discovered by Prof. R. W. Wood. The ideas developed by Bohr apply naturally to stimulation of the substance by electric discharge through its vapour; and as Prof. Wood remarked soon after his discovery of the phenomena, we have in Resonance Radiation, an entirely new method of stimulating a substance to emit radiation, and it would be highly interesting to consider whether Bohr's ideas would apply *en bloc* to this case as well. Some peculiar qualitative features of the phenomena, such for example, as the transformation of the Resonance spectrum into a band spectrum under certain conditions, seem to promise very much in throwing light on the problem of atomic structure.

§ 2. *On the Law of Spacing of the Resonance Lines.*

In a recent paper* in the *Philosophical Magazine* an attempt has been made by Dr. Silberstein, to explain the phenomena of Resonance Radiation, on the prin-

* Dr. Silberstein on *Fluorescent vapours and their magneto-optic properties* Phil., Mag., Sept. 1916.

ciples of classical mechanics. Dr. Silberstein considers, that the electron which emits the radiation, is a non-Hookean resonator *i.e.*, a system in which the restitutive force is not simply proportional to the displacement, the case being analogous to that of combinational tones in sound. Dr. Silberstein follows the analogy, and introduces a small term, depending not upon the square as in the case of sound, but upon the ϕ^{th} power of the displacement, where ϕ is nearly but not exactly equal to one, and thus writes his equation of motion as

$$y + n^2 y + \alpha y^{\phi} = e^{\text{int}}$$

The consequence of this assumption he finds to be, that the Resonance series should be characterized by constant frequency intervals. He gives in his paper a table, prepared from Wood's data, and thinks that his conclusions from the theory are supported by facts. But a critical examination of his own figures shows that the frequency intervals have a most decided tendency to decrease on the long wave-length side. There are indeed some irregularities, but it is probably due to the fact that he works out successive differences. Adopting a method of calculation, in which the effect of experimental errors is better minimised, I have prepared a table from Wood's data* for the frequencies of the components of the Resonance series of Iodine vapour, and I have found that it is not the frequencies themselves, but their square roots, that have constant intervals in this series.

* Wood. Phil. Mag., 24, p. 684.

TABLE.

1 Serial No. n .	2 Frequency. ν_n	3 $\frac{\nu_0 - \nu_n}{n}$	4 $\frac{\nu_0^{\frac{1}{2}} - \nu_n^{\frac{1}{2}}}{n}$
	$10^{10} \times$	$10^{10} \times$	$10^3 \times$
0	54937.5		
1	54279.0	658.5	141.0
2	53646.9	645.3	138.5
3	53013.0	641.5	137.0
4	52386.0	637.8	137.7
5	51759.0	635.7	137.6
6	51144.0	632.3	137.3
7	50523.0	630.7	137.4
8	49911.0	628.2	137.3
9	...	...	...
10	48696.0	624.1	137.2
11	48096.0	621.9	137.1
12	47493.0	620.4	137.2
13	46902.0	618.1	137.2
14	...	...	...
15	45726.0	614.1	137.1
16	45147.0	611.9	137.0
17	44562.0	610.3	137.1
18	43983.0	608.6	137.4
19	43419.0	606.2	136.9
20	42855.0	604.1	136.9

I have shown in columns 3 and 4 the intervals of the frequencies and the intervals of their square roots respectively; the former are by no means constant, decreasing systematically by about 9% down the column, while the latter are, except for the first few lines, remarkably constant. The attempt to explain the phenomena, on the principles of classical mechanics is apparently, therefore, not quite successful even as regards the spacing of the Resonance lines, not to mention other aspects of the question not considered by Silberstein. I propose in the present paper to suggest an explanation of the phenomena on the Quantum theory.

§ 3. On the Theory of the Stationary States.

It may be pointed out at the very outset, that we cannot account for the law of spacing of the components in the Resonance series, on Bohr's simple theory of spectral series. For the law that is characteristic of his theory, is of the Balmer type, the variable factor being $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and the frequency interval between two successive components in the series will be proportional to $\left\{\frac{1}{n_2^2} - \frac{1}{(n_2+1)^2}\right\} = \frac{2}{n_2^3}$ approximately. This quantity varies with n_2 , and the series will not represent anything like the resonance series, unless n_2 were as large as 400 or 500, in which case the orbit of the radiating electron must be of dimensions enormously large compared with molecular dimensions, being of the order 10^{-3} cm. It is apparent, therefore, that we must take into account the effect of the other electrons in the system on the radiating electron and consider

in greater detail the theory of the possible frequencies of radiations from an atomic system having a large number of electrons.

Bohr regards Light radiations to be due to electrons falling into a system which has lost one or more electrons from the outer ring. And as in the course of the reformation of the system it is extremely unlikely that two of the falling electrons are at any instant at the same distance from the nucleus, our problem becomes practically equivalent to determining the "stationary states" and the frequencies of the radiations of an electron in a system consisting of a few electrons revolving one in each ring round the rest of the molecular system. Let us also assume that any one of the electrons, not necessarily the outermost one, has a series of "stationary states" throughout the system, by passage from one to another of which, it can absorb or emit radiation of appropriate frequency.

On these general assumptions, we can derive a formula for the frequencies of the possible radiations from the system. Leaving out the radiating electron itself from reckoning, let us suppose that it lies at any instant between the orbits of the $(\tau_0-1)^{\text{th}}$ and the τ_0^{th} electrons (counted from the outer surface of the system). Then since an electron revolving in a circle, produces very little effect at a point inside its orbit, while for a point outside, the effect is nearly the same, as if it were placed at the centre of its orbit, the radial force on the electron, may be supposed to be due to an equivalent nucleus of charge Se where

$$S = \tau_0 + \phi(\nu) \quad \dots \quad \dots \quad \dots \quad (1)$$

$\phi(\nu)$ being the correction factor, which is a function

of r the radius of the orbit. The function will be definite, if the configuration of the other electrons is given. The force on the electron may hence be written

$$F = \frac{S}{r^2} e^2 \quad \dots \quad \dots \quad (2)$$

Let v be the velocity of the electron in its orbit. Then

$$\frac{mv^2}{r} = F = \frac{Se^2}{r^2} \quad \dots \quad \dots \quad (3)$$

and since the angular momentum of the electron is a multiple of $\frac{h}{2\pi}$, $mvr = \frac{\tau h}{2\pi}$, where τ is an integer. From these equations, we get,

$$r = \frac{\tau^2 h^2}{4\pi^2 m} \cdot \frac{1}{Se^2} = \frac{h\tau^2}{S} \text{ (say)}. \quad \dots \quad (4)$$

If W be the energy of the electron in any of its "stationary states"

$$W = \frac{1}{2}mv^2 = \frac{2\pi^2 e^4 m}{h^2} \cdot \frac{S^2}{\tau^2} \quad \dots \quad (5)$$

so that the possible frequencies of the radiations from the system are given by

$$\nu = \frac{2\pi^2 e^4 m}{h^3} \left\{ \frac{S_1^2}{\tau_1^2} - \frac{S_2^2}{\tau_2^2} \right\} \quad \dots \quad \dots \quad (6)$$

Even without knowing the exact nature of the function $\phi(\nu)$ in equation (1), it is obvious that the quantity S will increase as r increases, for the repulsion of the inner electrons becomes less, and that of the outer increases. Hence it is evident that, since

$$\frac{S_{\tau_2}}{\tau_2^2} - \frac{S_{\tau_2+1}}{(\tau_2+1)^2} < S_{\tau_2} \left\{ \frac{1}{\tau_2^2} - \frac{1}{(\tau_2+1)^2} \right\},$$

closer groups of lines are possible on this theory than on a simple series of the Balmer type. We may now consider what is likely to happen when light of a definite frequency ν excites the system; it will be

absorbed or, in general, will affect the system, if any one of the electrons in the system requires an amount of energy $h\nu$ to pass from any one of its stationary states to any other. Suppose for definiteness, that for the mercury green radiation this condition is satisfied by an electron in a stationary state between the orbits A and B of its neighbours (fig. 1), if it passes to

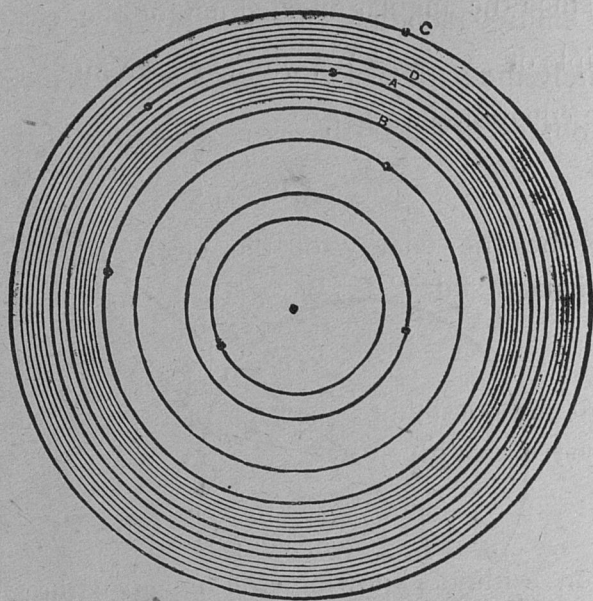


Fig. 1.

a stationary state between C and D orbits, of two other consecutive electrons. Then as soon as the light falls on the system, it will jump from the former to the latter state, absorbing the incident radiation. It then jumps back, emitting radiation. If it jumps exactly to its original orbit, the frequency of the emitted radiation will be the same as that of the incident one; but when it jumped, the configuration must have changed, so

that it is not likely that it jumps exactly to its original path. Let us assume that it jumps to one of the series of stationary orbits; then it is clear, that in the resulting radiation, there will be as many different components as the number of orbits. The frequency intervals of these components will obviously depend upon the difference in the function $\frac{S^2}{r^3}$, between the consecutive orbits. To determine the spacing of the lines in the emitted radiation, therefore, we must know more definitely the nature of the function $\frac{S^2}{r^3}$. We shall now work out, as an illustrative case, the nature of that func-

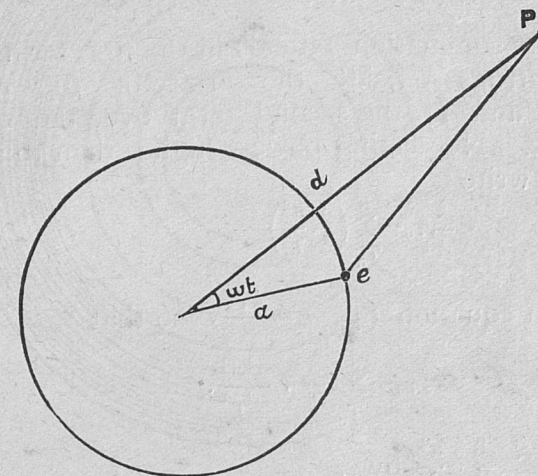


Fig. 2.

tion, on the assumption that the force exerted by a revolving electron is correctly represented by its time-mean value. Thus, if f be the mean force due to an electron revolving in a circle of radius a , at a distance d from its centre,

$$f = \frac{1}{T} \int_0^T \frac{e (d - a \cos \omega t)}{(a^2 + d^2 - 2ad \cos \omega t)^{\frac{3}{2}}} \delta t.$$

If $d < a$

$$f = \frac{e}{2\pi d} \int_0^{2\pi} \frac{1}{a} \left[P_0(\cos\theta) + \frac{d}{a} P_1(\cos\theta) + \dots \right] d\theta$$

$$= -\frac{e}{d^2} \left(\frac{1}{2} \frac{d^3}{a^3} + \frac{9d^5}{16a^5} + \dots \right)$$

If $d > a$

$$f = \frac{e}{d^2} \left(1 + \frac{3}{4} \frac{a^2}{d^2} + \frac{45}{64} \frac{a^4}{d^4} + \dots \right)$$

The function $\phi(r)$ becomes

$$\phi(r) = \sum_{\tau_0-1}^1 \left(\frac{1}{2} \frac{r^3}{a^3} + \frac{9}{16} \frac{r^5}{a^5} + \dots \right)$$

$$- \sum_{\tau_0}^N \left(\frac{3}{4} \frac{a^2}{r^2} - \frac{45}{64} \frac{a^4}{r^4} + \dots \right).$$

where the summation extends in the first term over all the electrons outside the orbit of the radiating electron, and in the second term over the electrons inside it. Neglecting powers higher than the square, we may write

$$S = \tau_0 - \sum_{\tau_0}^N \left(\frac{3}{4} \frac{a^2}{r^2} \right)$$

But from equation (4), $r = \frac{h\tau^2}{S}$, so that

$$S = \tau_0 - \sum_{\tau_0}^N \frac{3}{4} \cdot \frac{a^2 S^2}{h^2 \tau^4}.$$

Denote $\sum_{\tau_0}^N \frac{3}{4} \cdot \frac{a^2}{h^2 \tau^2}$ by c .

Then $\frac{Sc^2}{\tau^4} + S - \tau_0 = 0$.

Hence $S = -\frac{\tau^4}{2c} + \left\{ \frac{\tau^8}{4c^2} + \frac{\tau_0 \tau^4}{c} \right\}^{\frac{1}{2}}$

$$S^2 = \frac{\tau_0 \tau^4}{c} + \frac{\tau^8}{2c^2} - \frac{\tau^4}{c} \left\{ \frac{\tau^8}{4c^2} + \frac{\tau_0 \tau^4}{c} \right\}^{\frac{1}{2}}$$

now $c = \sum \frac{3}{4} \frac{a^2}{h^2 \tau^2}$ and will be of the order $\sum \frac{3}{4} \frac{\tau_m^4}{m^2}$, if τ_m corresponds to the orbit of the m^{th} electron, and

unless the electrons in the periphery of the atom are revolving in orbits extremely close to one another, c will hence be smaller than $\frac{\tau^4}{\tau_0^2}$. The first term under the square-root will be larger than the second, and we may write

$$S^2 = \frac{\tau_0 \tau^4}{c} + \frac{\tau^8}{2c^2} - \frac{\tau^8}{2c^2} \left(1 + \frac{2c\tau_0}{\tau^4} - \frac{2c^2\tau_0^2}{\tau^8} + \frac{4c^3\tau_0^3}{\tau^{12}} - \dots \right)$$

$$= \tau_0^2 - \frac{2c\tau_0^3}{\tau^4} + \dots$$

$$\frac{S^2}{\tau^2} = \frac{\tau_0^2}{\tau^2} - \frac{2c\tau_0^3}{\tau^6} + \dots = y \text{ (say).}$$

Now the frequency intervals between two successive components of the series of radiations must be proportional, on the present theory, to the difference in the value of this function y in any two consecutive "stationary states." We can approximately obtain the latter by differentiating y with respect to τ . Thus

$$\frac{dy}{d\tau} = -\frac{2\tau_0^3}{\tau^3} + \frac{12c\tau_0^3}{\tau^7}.$$

Let us reckon from a line in the series which corresponds to the path τ_1 and write for the successive lines $\tau = \tau_1 + n$. Then if ν_{n-1}, ν_n be the frequencies of two successive lines in the series of radiations.

$$\frac{1}{K} (\nu_{n-1} - \nu_n) = \frac{2\tau_0^2}{\tau_1^3} - \frac{12c\tau_0^3}{\tau_1^7} - \frac{n}{\tau_1} \left(\frac{6\tau_0^2}{\tau_1^3} - \frac{84c\tau_0^3}{\tau_1^7} \right)$$

$$- \frac{n^3}{\tau_1^3} (\dots) \dots \dots (7)$$

and can be written in the form

$$\nu_{n-1} - \nu_n = \alpha + \beta n + \gamma n^2 + \dots$$

If the coefficient α, β, γ etc. are all exactly zero, the resonance series will be characterized by constant frequency intervals. It does not seem possible, without

making further assumptions to determine their magnitude absolutely. But though it is unlikely that α, β, γ , all vanish, it can be seen that the successive coefficients will decrease rapidly. In the case of Iodine vapour, it was pointed out in the previous section that $\nu_{n-1} - \nu_n$ seemed to be constant. This relation is equivalent to

$$\nu_{n-1} - \nu_n = 2\nu_0\delta - 2n\delta^2,$$

where $\delta = \nu_{n-1} - \nu_n$. Thus $\nu_{n-1} - \nu_n$ is of the form $(\alpha + \beta n)$. It is also possible to show that the quantities α, β given by equation (7) are of the right order. Thus in the case of Iodine vapour $\frac{\alpha}{K}$ is about 2×10^{-3} and $\frac{\beta}{K}$ about 10^{-5} . Hence we shall have

$$\frac{2\tau_0^2}{\tau_1^3} - \frac{12c\tau_0^3}{\tau_1^7} = 2 \times 10^{-3}.$$

$$\frac{1}{\tau_1} \left\{ \frac{6\tau_0^2}{\tau_1^3} - \frac{84c\tau_0^3}{\tau_1^7} \right\} = 10^{-5}$$

Putting $\tau_0 = 2$, we get τ_1 to be about 13, and the diameter of the orbit of the electron will be of the order of 10^{-6} cm., which does not seem improbable, when account is taken of the fact that the vapour must be at a very low pressure to emit Resonance radiation.

It may be pointed out here, however, that since the equation $\nu_{n-1} - \nu_n = \alpha + \beta n$ is more general than the equation $\nu_{n-1} - \nu_n = \delta$, it seems only accidental, on the present hypothesis, that the latter formula holds very approximately in the case of Iodine vapour. In the case of Sodium vapour it has been found by the writer, that while there is no approach to constant frequency intervals, even the expression $\nu_{n-1} - \nu_n$ gives

a systematic deviation on one side, so that this form has perhaps no special significance.

One point still remains to be noticed. In a Resonance series ordinarily as many as 20 lines are observed, and we cannot certainly assume that there are as many orbits between two consecutive electrons of the system, to which it can jump. This difficulty is easily got over, if we assume that it need not jump from the same orbit every time, but from one of a series of orbits between C and D (fig. 1). It is obvious, then, that we can account for as many as 25 lines in the resonance series with only 5 orbits in each series; suppose, for example, the interval of the function $\frac{S^2}{\tau^2}$ is constant over successive orbits in each series, but the interval in one series is 5 times that in the other; then, we shall have 25 lines in the resonance spectrum at constant frequency intervals. It seems at first sight unlikely that the resonance series of Iodine vapour is made up of smaller series patched up like this; for, if it is so formed, the patching up must be very nicely adjusted, so as to make the series indistinguishable from a single continuous series. But certain observations of Wood*, regarding the Resonance radiation of Sodium vapour, seem to bear out the suggestions that have been put forth here. Wood has found that, as we go away from the resonance line, new lines appear, which do not belong to the old series, and new series start from where the old ones leave off. This is exactly what should be expected, if in the above illustration considered, the interval of

* Prof. R. W. Wood. Phil. Mag., 25, p. 588.

$\frac{S^2}{r^3}$ in one series of orbits is not an exact multiple of that in the other series. As a matter of fact, generally only 4 or 5 lines can be made out as forming one continuous series; and the whole resonance spectrum is made up of bits like these. Even in the case of Iodine vapour, a grouping suggested by Wood* from a consideration of the structure of these lines, seems to give additional evidence to the suggestions offered here.

§ 4. *Some Characteristics of Resonance Radiation.*

Coming back to the mechanism of resonance radiation, the main point in the hypothesis suggested here is that soon after the external radiation has begun to excite the system, if not even before, in the permanent configuration of the system itself, the electrons in the periphery of the atom are revolving one in each ring round the nucleus, and one particular electron absorbs and emits light by passing from one of a series of "stationary states" between two consecutive electrons to another series of states between another pair of consecutive electrons. The frequency intervals between successive components in the emitted radiation are proportional to the intervals of the function $\frac{S^2}{r^3}$ over consecutive "stationary states" in the same series, and, as is evident from the analysis given in the previous section, they will depend upon the configuration of the other peripheral electrons, both relative to themselves and relative to the nucleus. It is not to be expected, *a priori*, that this configuration will be the

* Wood. Phil. Mag., 26, p. 841.

same for all the molecular systems, and hence each molecular system may, in general, be expected to give its own Resonance series. We can more easily conceive, therefore, of the system giving a band-spectrum than a line-spectrum. Experimental evidence seems generally to support this conclusion, Resonance spectra being exhibited by very few substances, and by these, only under particular conditions of temperature and pressure; and they are either destroyed altogether, or give place to the band-spectrum under slight disturbing causes. But the fact that it does occur under some conditions, points to the conclusion that there are some stable or quasi-stable configurations which are characteristic of the substance and hence assumed by the falling electrons in all the systems at once. All configurations very near these quasi-stable configurations lead to these latter configurations, though a configuration very different from them may not do so, the case being perhaps something like that of the configurations that may be assumed by a system of floating magnets.

On this view, we find an easy interpretation of the fact that the resonance spectrum of Iodine vapour, is transformed into a band-spectrum, on rarefying the vapour; that is to say, that the same Iodine vapour, when it is at a lower pressure, gives under the same monochromatic excitation, an almost continuous spectrum, instead of the characteristic line spectrum. Wood*, who discovered this phenomenon, gives, however, a different explanation. He thinks that the band-spectrum is always produced, but is absorbed by the fluorescent gas itself when it is dense. He cites

* Wood. Phil. Mag., Nov. 1913.

as an evidence for this, an experiment tried by him, which was to pass the radiation from the rarefied vapour, through another tube containing cool iodine vapour. He found that the band-spectrum was completely absorbed, the resonance lines alone being left out. I will show presently, that the result of this experiment is not inconsistent with the views I have tried to develop. It may be pointed out, meanwhile, that apart from some objections that may be raised against his conclusions, his hypothesis does not lead us very far, and leaves it for further explanation—why from an almost continuous spectrum, only certain lines should be left out unabsorbed. Wood has indeed realised the difficulty, and has raised the question whether the same molecule emits both the resonance and the band-spectra, or only one, and whether in the vapour there are simultaneously systems emitting the two spectra independently. But the essential point about it seems to me to be, that it does not throw any light on the occurrence of the band-spectrum under apparently totally different conditions. Wood and Frank* have discovered that the resonance spectrum of Iodine vapour is transformed into a band-spectrum by the introduction of the gas Helium at a very low pressure. Indeed, the discovery was prior to the one previously mentioned; and Wood says nothing more on it than that it is due to the collision of the Helium molecules with the Iodine molecules. He regards the phenomena as a new effect of molecular collisions on the radiations from an atomic system, and has left it as a problem for the theoretical

* Wood and Frank. *Phil. Mag.*, Feb. 1911.

physicists to solve. The problem, so far as I know, has not been solved; but Wood has tried to find out whether the fact that Helium has a small affinity for electrons, has anything to do with its behaviour. The facts that have been observed, may be briefly set forth here and an explanation then suggested. Electro-negative gases, that is, gases which have strong affinity for electrons, completely destroy the fluorescence. Gases which have a weak affinity for electrons, like Helium, transform the line-spectrum into a band-spectrum, but this effect does not vary inversely as the affinity for electrons, for Neon, which has a much smaller affinity for electrons than Helium, shows scarcely so marked an effect as Helium in transforming this spectrum. The explanation is briefly this. Electro-negative gases pull out the outer electrons in the Iodine molecule (which are of course those responsible for the resonance spectrum), pull them either into their own molecules or at least so far away from the nucleus as to destroy the condition of absorption of the incident light. Gases like Helium with less affinity for electrons, do not pull these outer electrons so far as to destroy the absorption of the incident light, but far enough to destroy the quasi-stable configurations described above. Gases with absolutely no affinity for electrons, apparently do not produce any effect on the Iodine molecules or on the radiation from them.

I may now explain why the resonance lines are not absorbed, while the band-spectrum is absorbed * when passed through a tube containing cool Iodine vapour.

* See Wood. *Phil. Mag.*, Nov. 1913, p. 838.

The radiation of the resonance lines depends upon a particular quasi-stable configuration assumed by the system, under the action of the exciting light, and naturally, on our theory, can be absorbed by the system only when in that configuration. On the other hand, the band-spectrum is due to molecules in varying configurations, more or less distributed at random, and the vapour in its ordinary state will also generally contain such systems with no unique configuration for all the molecular systems. It is only to be expected, therefore, that the band-spectrum should be absorbed. This explanation is indeed analogous to that given by Bohr* for the fact that Hydrogen does not exhibit absorption lines corresponding to all its radiation frequencies.

We may now pass on to consider the structure of these resonance lines, and also the companion lines that are found in large number distributed throughout the spectrum. As it has been pointed out, it seems very unlikely that under any conditions the configurations of all the molecular systems are the same, though in the case of certain quasi-stable configurations at least most of the molecules must have the same configurations if the substance can emit a line spectrum. But even in the latter case, one would naturally expect some systems to remain "out of tune," and it is suggested here that the companion lines observed are due to these systems which are "not in tune" with the rest. - A great point in favour of this suggestion is that most of these lines are faint in comparison with the other lines, which is just what

* Bohr, Phil. Mag., July 1913.

we should expect, since only a few systems are supposed to be emitting them. Further, even in the other systems, if the adjustment is not perfect we should expect the resonance lines to be not quite homogeneous. Wood has found that the structure of these resonance lines is in fact very complex, the number of components in each line observable seeming to depend only on the resolving power used. He considers this to be due to the complex structure of the exciting mercury green line, but as he himself says, the structure of the resonance lines is much more complex, so that each exciting line gives rise to more than one line in the resonance lines or line-groups, as we may call them. Indeed, some of his observations on the complicated effects of changing the structure of the lines, lead us to doubt whether the phenomena of resonance radiation can ever be satisfactorily explained on the principles of classical mechanics.

Reference may now be made also to the point, in what way, if at all, resonance radiation is simpler than radiation excited by other means. Since we use a mono-chromatic exciter, Wood thought that in resonance excitation, we were, as we may call it, striking a single key in the key-board of the atom; and the comparatively simple structure of the resonance spectrum seemed to give weight to this conjecture. But we now know that the emitted radiation is so simple, only under certain conditions; while, under other conditions, the same mono-chromatic exciter gives rise to a band-spectrum. On the views set forth in this paper, the simplicity of the resonance

spectrum is due not to the nature of the exciter but to the nature of the system, to the fact that it has some quasi-stable configuration of the nature described already. In fact, the assumption of such configurations is necessary, even in the case of electrical excitation; Hick's* four "Sequences," for example, require four such definite configurations.

§ 5. Röntgen Spectra.

We may now pass on to consider another set of phenomena, which we may term also resonance radiation; when X-rays fall upon a substance, they excite it to something like resonance, when their frequencies are near certain values characteristic of the substance. The resulting radiation from the substance is termed, as we know, the Characteristic radiations, and consists of at least three series of lines, the K, L and M series as they are called. In an epoch-making discovery, Moseley has shown that the characteristic radiations from different elements were related in a simple manner to their relative positions in the periodic table of elements, and more definitely that the frequencies of the K-Characteristic radiations of the elements are represented by the equation

$$\nu = K(N-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

where N is the atomic number and K is the Rydberg constant for Spectral series. He wrote it in this form to bring out the analogy with the Balmer formula, and tried to interpret the formula on Bohr's theory by supposing that the radiations were due to the vibra-

* Hicks, Proc. Roy. Soc., **83**, p. 226.

tions of a ring of four electrons, immediately surrounding the nucleus. But it has been pointed out by Nicholson* and Bohr†, that this is not the correct interpretation, since we will have to assume that several quanta are emitted at the same time. Ishiwara‡ has pointed out that the lines in the Röntgen spectra can be accounted for, if Bohr's spectral formula be altered to

$$\nu = K \left\{ \frac{(N-c_1)^2}{(\mu_1 + \mu_1)^2} - \frac{(N-c_2)^2}{(\mu_2 + \mu_2)^2} \right\}.$$

Remembering that the region of excitation in the molecular system is now the innermost ring of elec-

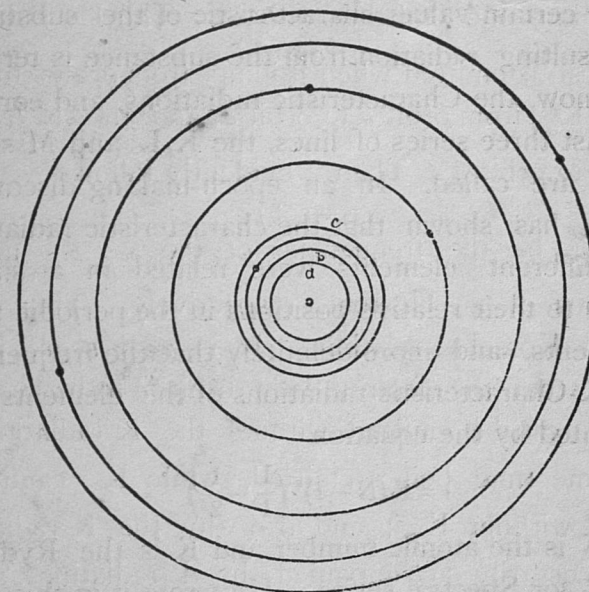


Fig. 3.

* Nicholson, Discussion on the Structure of the Atom, March 19, 1914. Proc. Roy. Soc., vol. 90.

† Bohr, On the structure of the Atom and the Quantum Theory of Radiation, Phil. Mag., Sept. 1915.

‡ Jun Ishiwara, Proc. of the Tokyo Math. Phys. Soc., Ser. 2 vol. 9, p. 160. (July 1917).

trons instead of the outermost, this form can be derived from the general formula obtained in this paper for the frequencies of the radiations from an atomic system, by giving a suitable value to the function $\phi(r)$. A detailed conception of the mechanism of emission of these radiations is also possible. The electron which emits the radiations (very probably the first from the nucleus) is supposed to be in one of its stationary states b (fig. 3) inside the orbit of the second electron. Under proper excitation, it is supposed to jump out to an orbit c just outside the second electron absorbing energy, and then back again to an orbit near its previous one emitting radiation. The L_α radiation, I suggest, is emitted when it jumps from orbit c to orbit b ; the K_α , from orbit b to orbit a , and the K_β , from orbit c to orbit a . Then it evidently follows

$$\nu_1 = \nu_2 + \nu_3$$

if ν_1, ν_2, ν_3 , be the frequencies of the K_β, K_α , and L_α radiations of an element, respectively. This is, of course, Kossel's relation. It is also seen, if we assume that the electron is in orbit b to begin with, why L_α can be emitted without the K-radiations at the same time being emitted, while K_α cannot be emitted without K_β ; and also why the K-radiations are necessarily accompanied by L-radiations. The other lines in the Röntgen spectrum can similarly be accounted for by considering some more orbits of the radiating electron.

The secondary corpuscular radiation, which is found to invariably accompany the emission of Characteristic radiations, may be supposed to be due to the collision

of the radiating electron with the second electron in one of its passages across its orbit, and the consequent dislodgment of the latter. One remarkable result will follow from this supposition, that the velocities of the corpuscular rays should be the same for both the K and L Characteristic radiations. This has really been found to be the case by Barkla and Shearer*, who have noted it as a remarkable fact in view of the general supposition, that the radiations have their origin in two different rings of electrons.

§ 6. The general law of Spectral Series.

Before concluding the paper, I will show you how from the atomic structure suggested in this paper a general formula for spectral series can be obtained taking into account all possible regions of excitation. As it has been shown, the frequencies of radiation from an atomic system are given by

$$\nu = K \left\{ \frac{S_1^2}{\tau_1^2} - \frac{S_2^2}{\tau_2^2} \right\}$$

Now $S = \tau_0 + \phi(r)$ where r is the radius of the orbit of the radiating electron, and

$$r = \frac{k\tau^2}{S}$$

so that

$$S = \tau_0 + \phi\left(\frac{k\tau^2}{S}\right).$$

on solving which, we may express S as an explicit function of τ of the form

$$S^2 = \psi_1(\tau)$$

the suffix being introduced to take account of the fact

* Barkla and Shearer, "On the Velocity ~~and~~ electrons expelled by X-rays," Phil. Mag., Dec. 1915.

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that the function will change in passing from between one pair of consecutive electrons to another. Thus

$$\nu = K \left\{ \frac{\psi_r(\tau_1)}{\tau_1^2} - \frac{\psi_s(\tau_2)}{\tau_2^2} \right\}.$$

This is of the same form as Ritz and Rydberg's general formula, though, as has been indicated, it also represents lines in high frequency spectra.

One point seems worthy of notice. An examination of any general law of spectral series shows us that the functions ψ_r and ψ_s are in actual cases found to be different; in Hicks law of spectral series, for instance, the lines in the spectra are never got from the same "sequence," but from two different sequences, the integral parameter in one of which is given a constant value while that in the other is allowed to vary. This would indicate that the radiations are due to an electron jumping into an orbit between two consecutive electrons in the system from one of a series of orbits outside the two electrons; and it may be remembered that we made the same assumptions in explaining Resonance radiation. It is indeed difficult to explain the fact stated here, if we consider light radiation to be due to the outermost electron, or rather to an electron which remains always the outermost in the system.

In conclusion, it may be pointed out that only the broad outlines of the subject have been touched upon in this paper, so as just to show what light the hypothesis suggested here throws upon the observed phenomena of radiation. The further development, especially with reference to quantitative relationships, is a matter for subsequent investigation.

December 15, 1917.

47

Home Education Dept

CNo 4 A. from the Indian Association for the Cultivation

of Science & 30.12.17
 n 1634 n D.O. no 413 of 18.1.18
 [Subject - application of Mr. R. T. K. Chinmaya
 nandanam for a post in the I. E. S.]

Please see the Director's letter on pages 95 to
 of c. f. ~~Submitted that~~ Mr. Chinmayanandan
~~has already been~~ informed that there is no vacancy
 in this Presidency for which his attainments
 and aptitudes would specially fit him.
~~and that he should submit his application to the Govt
 of India through the Govt of Bengal.~~

2. With reference to para. 1 of the
 application (page 3, cf.) it may be noted that
 the proposals for filling up the existing vacancies
 in the I. E. S. have already been submitted
 to the Govt. of India - fo. no 4^x-5 of 3.1.18

x put up in another file

3. It is for consideration whether
 a formal reply from Govt. should be
 sent or whether the papers may be
 recorded.

We may reply with reference
 to the application made to
 us and direct the applicant
 to apply to the Govt. of India.

11/1
 26/1/18

21.1.18

Please draft my 2/2/18

The draft as revised is for approval.

by
2-2-18
ms
8/10/18

Q
4/2/18

W 5/2/18

48

J. E. S.

READ AGAIN G.O. No., dated 191 .

” ” ” ” ” ” 191 .

● READ ALSO:—

C. No. 4A, From the Sri Taraknath ~~Path~~ Peth, No. 11, 84, dated 30.12. 1911

C. No. 911, From the Hon. Member, Professor Physics, Calcutta University

663A D.D. I. Ref. no. 413 18-1-1918

” ” ” ” ” ” ” 191

ORDER.

mioc.

No. 139, dated 7.2 1918

With reference to his application dated the 30th December 1917, Mr R. K. Chinmayananda is informed that there ~~was no vacancy in the Educational Deptt of~~ ^{reference} this Residency for which ~~his~~ ^{his} attainments and ~~qualifications~~ ^{qualifications} would specially fit ~~him~~ ^{him} and that ~~he~~ ^{that he} ~~may~~ ^{should} apply to the ~~Government of Madras~~ ^{Government of Madras} Govt. of India for the inclusion of his name in the list of candidates suitable for appointment to the D. E. S.

To Mr R. T. K. Chinmayandam
thru the Sir Taraknath Palit
Professor of Physics, Calcutta University
(Orissa).

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