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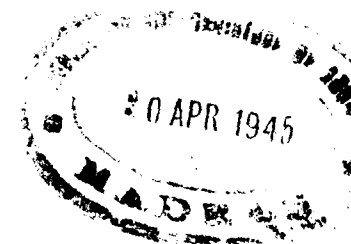
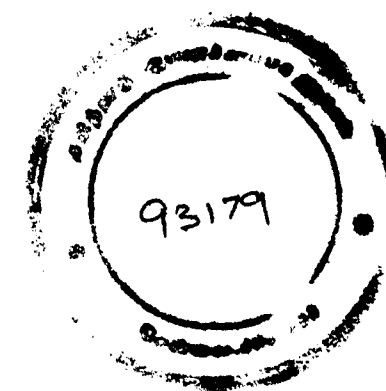


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A NOTE ON THE SEX-RATIO IN MORTALITY

By

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A study of the sex-ratio in mortality reveals certain interesting features of the operation of the Laws of Nature. Lack of adequate data is the chief obstacle for a scientific study of this problem. Vital registration has not yet fully developed in India to provide correct figures of deaths by sexes in every region. A study extending over a long period can be attempted with reliable results only for certain specific regions. In a previous paper the general trend for a period of 15 years, (1926-1940) and the seasonal variation of crude death rate in the City of Madras was undertaken.^(a) In this paper a study of the sex-ratio in deaths is made for the City of Madras for the same period of 15 years.

The required data of deaths by sexes for this study is taken from the weekly reports published by the Corporation of Madras. As pointed out in the previous paper there may be omissions and errors in the published figures.^(b) As the 7-day—period is too short to observe the effect of seasons, months of four weeks duration each are formed, giving 13 months to the year.^(c) The sex-ratio is obtained by taking the number of female deaths per hundred male deaths for all the months considered. Table I shows the figures of sex-ratio of all months for the 15 years. The last column and row indicate the mean yearly and monthly averages.

The averages in the last row and column are all less than 100, and the average for the whole period is 95.22. This clearly

(a) Rajalakshman, D. V.—'Mortality in the City of Madras—A study of Seasonal Variation—1926-'40'—Journal of the Madras University, Vol. XV, No. 2, p. 199—213.

(b) Ibid—p. 199.

(c) The corresponding calendar months of the Civil Year are shown in, Ibid—p. 200.

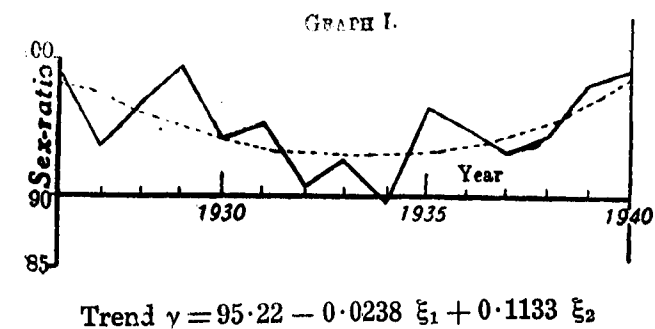
TABLE I.

Monthly figures of the sex-ratio in Deaths.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Average.	
1926	..	93.81	94.65	97.14	104.67	105.04	98.80	88.06	90.83	107.80	100.54	115.85	95.77	94.06	99.00
1927	..	95.80	85.94	91.27	91.46	92.53	94.73	87.47	96.55	104.73	103.57	87.16	92.87	95.75	93.83
1928	..	103.21	84.12	94.33	95.05	87.64	94.63	96.77	97.07	106.33	104.51	98.91	97.86	97.84	96.79
1929	..	92.42	99.78	94.48	97.32	100.87	108.11	102.50	92.50	100.95	107.65	100.35	97.99	98.56	99.50
1930	..	91.94	88.34	88.82	97.96	107.53	93.48	100.13	89.19	91.70	99.73	93.88	95.99	86.33	94.23
1931	..	99.11	94.23	89.42	92.06	92.31	102.76	96.24	93.19	106.23	94.23	89.23	93.58	95.92	95.27
1932	..	90.61	93.92	96.30	83.33	84.45	93.81	93.60	90.49	89.57	87.98	93.42	91.49	90.85	90.76
1933	..	89.20	96.85	93.44	99.36	95.83	91.68	96.39	93.99	94.09	87.07	84.68	93.10	87.33	92.54
1934	..	86.13	92.39	90.37	100.12	85.86	95.05	89.68	93.06	85.28	89.04	83.50	91.26	87.69	89.96
1935	..	101.06	87.80	97.14	98.06	99.05	104.02	90.30	105.75	91.88	100.00	94.13	95.41	91.42	96.62
1936	..	99.57	96.78	91.17	86.32	102.47	88.38	90.80	100.54	91.61	101.98	96.58	91.73	97.88	95.06
1937	..	93.61	84.62	93.30	89.09	92.62	97.43	92.79	92.49	102.78	97.48	92.49	89.84	92.18	93.13
1938	..	101.89	92.49	90.99	90.85	93.65	98.71	94.66	105.39	93.14	84.38	96.84	95.97	90.27	94.56
1939	..	89.87	93.37	103.34	108.63	97.23	98.17	99.33	97.21	100.76	99.70	99.51	93.32	94.60	98.08
1940	..	99.48	95.84	103.31	95.36	99.68	95.73	100.99	100.00	97.74	103.07	102.86	100.83	92.40	99.02
Average		95.18	92.07	94.32	95.31	95.78	97.03	94.65	95.88	91.64	97.40	95.29	94.47	92.87	95.22

indicates that the male deaths invariably exceed the female deaths. In a study of the sex-ratio in births for the City of Madras the corresponding phenomena of the excess of male births over the female issues is recorded by Mr. N. T. Mathew.^(a) It appears as if nature tries to maintain the equilibrium in sexes by off-setting the high male death rate by an excess in births. The low sex-ratio in mortality is not only peculiar to the City of Madras but is noted as a general feature of deaths. "Women generally have lower death-rate than men. This is brought out clearly by comparing the number of males and females surviving at different ages, as given in the life tables for different countries."^(b) Also "if we examine the susceptibility of women to particular diseases we find the same condition. Women seem to have greater resistance than men to all the more important diseases; perhaps they are less exposed than men."^(c) No specific cause can be attributed to the comparatively low mortality noted in females.

The yearly averages shown in the last column of Table I, indicate fluctuations without any definite trend. However a second degree polynomial is fitted to these averages, and the parabolic trend together with yearly averages are shown in Graph I.



The coefficients of both ξ_1 and ξ_2 are prominent, indicating a trough shaped trend as seen from the graph. No specific reasons can be given for the trend shown by the polynomial as the sex-ratio in mortality in any given year is influenced by innumerable factors.

(a) Mathew, N. T. 'The influence of seasons on human production'—Sankhya, Vol. V, p. 261—268 (1940-'41)—See p. 265.

(b) Warren S. Thompson—'Population problems'—p. 150.

(c) Ibid—p. 151.

As there are variations both in yearly and monthly averages, the method of the analysis of variance is adopted and the significance of the variation is tested by means of Fisher's z-transformation.

TABLE II.

Source.	Degrees of freedom.	Sum of Squares.	Mean Square.	Z	5 %	1 %
Between means of years.	14	1,588.75	113.48	0.7293	0.2682	0.3742
Between means of months.	12	476.28	39.69	0.2041	0.2804	0.3908
Interaction.	168	4,433.80	26.39			
Total.	194	6,498.83	33.50			

From the table the variation among years is definitely significant. Although seasons bring about a slight change in the variation in monthly sex-ratio (as the mean square for monthly variation is greater than the mean square of total variance), still it is not significant. This can be expected as there is no remarkable variation in the averages shown in the last row of table I. The lack of prominent seasonal variation in sex-ratio is also observed for births in the City of Calcutta.^(a) As pointed out by the authors "the sex-ratio is clearly a deep-seated biological constant which is not easily affected by environmental changes." This reason which is applicable to births can also be given for deaths. For the City of Madras the sex-ratio in births is noted by Mr. Mathew to exhibit a fairly prominent seasonal variation.^(b)

Seasonal indices are formed by the elimination of trend to determine the extent of the response of sex-ratio to seasonal variation. The monthly average figures are converted into percentages of the total average and the linear trend, which is very small, as already

(a) Kedarnath Das and P. C. Mahalanobis—'A preliminary note on the rates of maternal deaths and still births in Calcutta'—Sankhya, Vol. I, p. 215—230. See p. 230.

(b) Mathew, N. T.—Sankhya, Vol. V, p. 265.

seen, is eliminated^(a) from these percentage to give the seasonal indices. But as the trend is not linear, the indices thus obtained have to be adjusted to give a total of 1,300. Column (2), of Table III gives the monthly average figures of sex-ratio (noted in the last row of table I), Column (3) the percentages of the total average (95.22), Column (4) the seasonal indices eliminated for linear trend and Column (5) the seasonal indices adjusted for the total.

TABLE III.

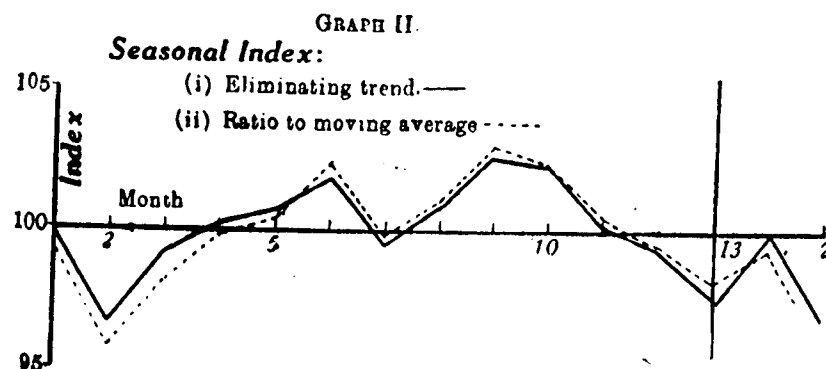
Month.	Sex-ratio.	Percentage Index.	Adjusted for trend.	Adjusted for total.	Ratio to moving average.	Corrected Seasonal Index.
1	2	3	4	5	6	7
1	95.18	99.96	99.960	99.947	99.082	99.357
2	92.07	96.69	96.692	96.680	95.508	95.773
3	94.32	99.05	99.054	99.041	97.904	98.176
4	95.31	100.09	100.096	100.083	99.581	99.857
5	95.78	100.59	100.598	100.585	100.145	100.423
6	97.03	101.90	101.910	101.897	102.086	102.369
7	94.65	99.40	99.412	99.399	99.458	99.734
8	95.88	100.69	100.704	100.691	100.690	100.969
9	97.64	102.54	102.556	102.543	102.699	102.984
10	97.40	102.29	102.308	102.295	101.989	102.272
11	95.29	100.07	100.090	100.077	100.090	100.368
12	94.47	99.21	99.232	99.219	99.191	99.466
13	92.87	97.53	97.554	97.542	97.979	98.251

The seasonal indices are also formed by the method of ratio to moving averages where linearity is assumed only for the logarithms of the actual numbers.^(b) The indices obtained by this method are shown in Column (6) of table III and Column (7) gives the

(a) The monthly trend to be adjusted is $\frac{.0238}{13} \times \frac{100}{95.22} = -0.002$.

(b) For the detailed proof of the formation of seasonal indices by the method of ratio to moving average, see, appendices I & II, of the Paper on Mortality—Journal of the Madras University, Vol. XV, No. 2, p. 207—213.

corrected indices after adjusting the total to 1,300. The corresponding graphs of the indices shown in Columns (5) and (7) are represented in Graph II.



The seasonal variations show the same tendencies by both the methods except for slight differences in intensity. The graph indicates three troughs in the 2nd, 7th and 13th months. Commencing from the end of February a steady rise is noted till the middle of June and then a sudden fall till the 15th of July followed by a rise reaching the maximum in September. Then a steady fall is noted till the end of December followed by a slight rise in January and then a fall in February. In general the female deaths may be taken to be normal from 4th to 11th month (i.e., from the end of April to the end of October) and below normal from 11th to 4th month (i.e., from November to April). The seasonal variation here is not seen to exhibit any marked relation to climatic conditions. But the seasonal variation, though not so prominent, seems to correspond a great deal to the seasonal variation in sex-ratio in births already investigated for the City of Madras.^(a) A high mortality rate in men appear to co-exist with high male birth-rate and vice-versa. It is also interesting to note that in a previous investigation regarding births in the city for 1926-1930, it was shown that sudden changes in temperatures, if they are sufficiently

(a) Mathew, N. T.—Sankhya, Vol. V, p. 265—See the monthly average figures given in the last row of table 3—These figures indicate a great negative correspondence to the graph of sex-ratio in deaths. This is due to the difference in the manner of computing the sex-ratio. Mr. Mathew took the percentage of male births to females whereas the percentage of female deaths to male deaths is adopted as a measure of sex-ratio in this study.

wide, affect the sex-ratio by lowering it (a fall in female births).^(a) Also the seasonal variation in sex-ratio in deaths has some similarity to the seasonal variation in birth-rate.^(b) High birth-rate is associated with high female deaths. This may possibly be due to the high maternal mortality and of mortality of women of child bearing age.

From this study of the trend and seasonal variation in sex-ratio in the mortality of the City of Madras it can be seen that there is no visible definite trend and the seasonal variation is not very prominent.

(a) Subramanian, M. K.—'Temperature as a factor in sex-determination in man'. Jour. Asiatic Society, Bengal (New Series) Vol. 29, pages 289—306 (1933).

(b) N. T. Mathew—Sankhya, Vol. V, p. 262—See the last row of table 1 giving the average monthly birth rates.

ON BINARY RELATIONS, CONGRUENCES AND
HOMOMORPHISMS

By

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Introduction.—Binary relations on a set have been studied by Tarski, Dubreil, Birkhoff, Ore and others. But a comparative study of the lattice-structures of the families of different types of relations has not been undertaken. I was led to make such a study in the course of developing a theory of congruences and homomorphisms for partially ordered sets, (which formed the subject matter of a thesis recently accepted for the M.Sc., Degree by the Madras University).

The method I had developed, in the thesis, for attacking the congruence theory for partially ordered sets could be applied with equal facility for other algebraic structures. This method I present here in its general form. (The study of congruences and homomorphisms for partially ordered sets—as given in the thesis—I shall deal with in a subsequent paper).

There is a simple correlation between 'equivalence relations' on a set, K , and those on the 'quotient', K/E , of K by a fixed equivalence relation E on it, (cf. Th. 3); using this, and the notion of an *induced* operation for K/E , when E is a 'congruence' relative to an operation, θ , I have been able to obtain an elegant generalisation—(cf. Th. 4)—of the following well-known Isomorphism theorem of Group Theory: if N is a normal subgroup of a group G , and $\bar{N}$ a normal subgroup of $\bar{G}=G/N$, the set-union of the residue classes of G modulo N which are in $\bar{N}$ is a normal subgroup, N^1 , of G such that G/N^1 and $\bar{G}/\bar{N}$ are isomorphic. This generalisation forms the main result in this paper.

I wish to take this opportunity to express my deep and sincere gratitude to Dr. R. Vaidyanathaswamy, Head of the Mathematics Department, for his kind help in the preparation of this paper.

CHAPTER I. BINARY RELATIONS ON A SET

§ 1. Binary relations on K and subsets of K^2 .

(1.1) A 'binary relation', R , on an abstract set, K , is a connective, or relation, relating certain ordered pairs of elements of K . When the pair (x, y) is connected by R we write ' xRy ' and say " x is in the relation R to y ".

If we denote by K^2 the set of all ordered pairs of elements chosen from K , it is clear that each relation R on K determines a unique subset of K^2 , namely: the set of pairs (x, y) for which xRy is true. This subset also determines R completely, since we consider relations only extensionally.

(1.2) Using the bi-univocal correlation thus established between the family, $B(K)$, of all binary relations on K and the family, $S(K^2)$, of all subsets of K^2 , we can extend to relations some of the familiar-notions of set-algebra as applied to $S(K^2)$. Thus we are led to the following definitions:

(1) The 'Universal-', 'Null-', and 'Identity-' relations— U , N and I —(which correspond, in order, to the whole set K^2 , the null set and the subset of repeated pairs of the form (x, x)), are defined by:—

xUy is true for all x, y from K ;

xNy is false for all x, y from K ;

and xIy holds, for x, y from K , if and only if $x=y$.

(2) We say " R is contained in S ", and write $R \subset S$, (in analogy to set-inclusion among subsets of K^2), where R, S are from $B(K)$, if xRy implies xSy for all x, y from K . $R \subset S$ and $S \subset R$ imply that R and S are (extensionally) identical; we then write $R=S$. Evidently $N \subset R \subset U$, whatever R be from $B(K)$.

(3) The 'negative', R' , of a relation R on K , (which is the analogue of the set complement in $S(K^2)$ of any element), is defined by:— $xR'y$ is true if, and only if, xRy is false. Evidently $(R')' = R$.

(4) For any family, F , of relations, $\{R\}$, "the Union ΣR - $R \in F$ ", and "the Intersection πR - $R \in F$ ", (which correspond to the set-union and set-intersection of a family of subsets of K^2), are defined as follows:—

$x\Sigma Ry$ holds if, and only if, xRy holds for *some* R in F ;
 $x\pi Ry$ holds if, and only if, xRy holds for *all* R in F .

For a two-element set, $F=\{R, S\}$, we also denote the Union by $R+S$, and the Intersection by $R.S$.

Finally we consider two important operations on relations which have no set-theoretic analogues.

(5) "The inverse", R^{-1} , of a relation R is defined by: $xR^{-1}y$ holds if, and only if, yRx holds. Evidently $(R^{-1})^{-1}=R$, $(\Sigma R)^{-1}=\Sigma R^{-1}$, $(\pi R)^{-1}=\pi R^{-1}$, and $R \subset S$ implies $R^{-1} \subset S^{-1}$.

(6) "The relational product", $R \times S$, of two relations R, S is defined by: $x(R \times S)y$ is true if, and only if, an element z can be found in K for which both xRz and zSy are true. (This product should not be confused with $\cdot$ defined in (4); whereas $\cdot$ is commutative this product, $\times$, is not). It is evident that $\times$ is associative; that is, $R \times (S \times T) = (R \times S) \times T$. So the relational product, $R_1 \times R_2 \times \dots \times R_n$, of a finite sequence of relations is uniquely defined by: $x(R_1 \times \dots \times R_n)y$ is true if, and only if, a sequence of elements $x_1, x_2, \dots, x_{(n-1)}$ can be found in K satisfying $x R_1 x_1, x_1 R_2 x_2, \dots, x_{(n-1)} R_n y$. Also one can easily verify that $R \subset S$, and $R^* \subset S^*$ imply $R \times R^* \subset S \times S^*$.

On the basis of the above definitions we next classify the relations into types.

§ 2. Special types of binary relations.

(2.1) A binary relation, R , on a set K is defined to be:

- (i) "reflexive" if $I \subset R$; that is, if xRx holds for all x ;
- (ii) "symmetric" if $R=R^{-1}$; that is, xRy implies yRx for all x, y from K ;
- (iii) "transitive" if $R \times R \subset R$; that is, for any x, y, z from K , xRy and yRz imply xRz ;
- (iv) "a quasi ordering relation" if it is reflexive and transitive;
- (v) "an equivalence relation" if it is reflexive, symmetric and transitive;
- (vi) "an ordering relation" if it is reflexive, transitive and $R \cap R^{-1} = I$; that is, xRy and yRx imply $x=y$, for all x, y of K .

(2.2) Some immediate deductions regarding these types are:

- (1) U belongs to the first five types and I to all the six types; N is only symmetric and transitive.

(2) The inverse of a relation belonging to any one of the six types also belongs to the same type.

(3) The negative of a symmetric relation is symmetric.

(4) The Union of any family of reflexive (or symmetric) relations is also reflexive (symmetric).

(5) The Intersection of any family of relations, all belonging to any one of the six types, also belongs to the same type. (The proof of this for ordering relations follows from an earlier result, namely: $(\pi R)^{-1} = \pi R^{-1}$, $(R \cap F)$, for any subset F of $B(K)$).

(6) If R and S are both reflexive relations, then so is $R \times S$.

(7) If S is a reflexive relation and R any relation then $R \subset R \times S$, and $R \subset S \times R$.

(2.3) The equivalence relations on K are connected in a simple manner with the partitions of K into non-null, mutually-disjoint subsets. For, given any such partition, p , a unique equivalence relation $E_p = E(p)$, can be defined in terms of p by saying xEy holds if, and only if, x and y belong to one and the same subset of K into which p partitions K . While, conversely, any equivalence relation E is the equivalence, $E(p)$, corresponding, as above, to a partition p of K ; for, if we define $E(x)$ —for any element x of K —to be the set of elements y of K satisfying xEy , the distinct subsets of the form $E(x)$, as x ranges over K , evidently form such a partition p of K .

Under the above correlation between equivalence relations and partitions, the relation $\subset$ among equivalences corresponds to the relation of one partition being a refinement of another. Given an equivalence relation E on K , we call the subsets— $E(x)$ —of K , into which the associated partition p splits up K , "the cosets of K relative to E ", and the family, K/E , of such distinct cosets we call "the quotient of K by E " (by formal analogy to quotient structures in Algebra).

We next pass on to a short survey of the theory of partially ordered sets.¹

1. A detailed treatment of the theory is to be found in MacNeille's paper-of. (4) of Bibliography. The proofs of most of the results stated in §3 are also to be found in that paper.

§ 3. Partially Ordered Sets.

(3.1) An abstract set K with an associated ordering relation " $<$ " is called a "partially ordered set". The inverse " $>$ " of the relation " $<$ " is also an ordering relation on K ; and $x < y$, $x > y$ imply $x = y$, for all x, y of K . An element 0 , (1), of the partially ordered set K is a *zero*, (*a one*), of K if $0 < \text{every } x \text{ of } K$ ($1 > \text{every } x \text{ of } K$). If such a zero or one exists it is necessarily unique. Together 0 and 1 are called the *units* of K .

The *sum*, (the *product*), of a non-null subset A of K is defined to be an element s , (p), of K satisfying the conditions: each element of A is $< s$, ($> p$) and if each element of A is $<$, ($>$), an element z of K , then $s < z$, ($p > z$). It is easily verified that if such a sum or product exists it is unique. We denote the sum, s , and the product, p , also by Σa , $a \in A$, and πa , $a \in A$, respectively. When A consists of two elements a and b we write $a + b$ and $a \cdot b$ for s and p .

(3.2) By restricting a partially ordered set to be closed for combinations of the above operations we obtain special types of ordered sets; thus we have:

(1) "an additive system", ("a multiplicative system"), is an ordered set in which every *finite* subset has a sum, (has a product);

(2) "a lattice" is both an additive system and a multiplicative system;

(3) "a complete additive—", ("a complete multiplicative—"), system" is an ordered set in which *every* subset has a sum, (product).

(4) "a complete lattice" is both a complete additive system and a complete multiplicative system.

A lattice is said to be

"modular" if, (i) for any three elements a, b, c in it, $a > b$ implies

$$a \cdot (b + c) = b + a \cdot c;$$

"distributive" if, (ii) for any three elements a, b, c in it,

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

The laws (i) and (ii) imply, and are also implied by, their respective duals (i)' and (ii)' given below:

$$(i)' \text{ for any } a, b, c, a < b \text{ implies } a + b \cdot c = b \cdot (a + c);$$

$$(ii)' \text{ for any } a, b, c, a + b \cdot c = (a + b) \cdot (a + c).$$

(5) "A Boolean algebra" is a distributive lattice with units 0 and 1 , in which to each element c , there is a "Boolean complement", c' , satisfying the conditions: $c + c' = 1$, $c \cdot c' = 0$. This complement is necessarily unique. Also $a'' = (a')' = a$; and, for any subset A of the Boolean algebra, $(\Sigma a)' = \pi a'$, $a \in A$, $(\pi a)' = \Sigma a'$, $a \in A$, (provided these sums and products exist).

(6) "A complete Boolean algebra" is a Boolean algebra which is also a complete lattice.

(3.3) For the study of the lattice structure of relations later, I give now a few results regarding the structures defined above.

(1) A complete lattice always has a 0 and a 1 . And any complete multiplicative, (complete additive), system containing a *one*, (*a zero*), is a complete lattice.

(2) If a subset K' of a complete multiplicative system K contains the product in K of each subset of K' it is also a complete multiplicative system (when ordered by the same ordering relation as K); K' is, further, a "subsystem of K for unlimited multiplication"—that is, the product of *any* subset of K' is the same in K' as in K .

(3) If K is also a complete lattice in (2) above, and if K' contains further the one, 1 , of K , then K' is also a complete lattice with 1 as its one. (As before K' is a subsystem of K for unlimited multiplication, but not, in general, for addition; we can only assert that the sum of any subset of K' in K' is $>$ its sum in K).

§ 4. The Boolean algebra $B(K)$, and the lattices of special binary relations on K .

(4.1) Any family of sets can be partially ordered by the relation of "set-inclusion". When the family, $S(L)$, of all subsets of a set L , is so ordered it is a complete Boolean algebra. The whole set L , and the null set are its one and zero; the set-union and the set-intersection of any family of subsets give the sum and product in the Boolean algebra; and the set-complement of any set satisfies the conditions for the Boolean complement.

If, in particular, we take K^2 instead of L , and use the correlation, established in § 1, between $S(K^2)$ and the family $B(K)$ of all binary relations on K , we are led to the following conclusions:

(4.11) The relation $\subset$, defined in 1.2, partially orders $B(K)$; when so ordered $B(K)$ is a complete Boolean algebra with U and N as its one and zero; the union and intersection of relations (as defined in 1.2) constitute their sum and product respectively; and the negative of a relation takes the part of the Boolean complement of the relation.

(4.2) We consider next the structure of the subfamilies of special binary relations defined in §2. Denoting, in order, by $R(K)$, $S(K)$, $T(K)$, $Q(K)$, $E(K)$ and $O(K)$ the families of all reflexive, symmetric, transitive, quasi-ordering, equivalence and ordering relations on K , we shall prove the following:

Theorem 1: (a) $R(K)$ and $S(K)$ are complete Boolean algebras, and each is a subsystem of $B(K)$ for unlimited addition and multiplication. U is the one of each, while I and N are their respective zeros.

(b) $T(K)$, $Q(K)$ and $E(K)^2$ are complete lattices, and each is a subsystem of $B(K)$ for unlimited multiplication. U is the one of each, while N is the zero of $T(K)$, and I the zero of $Q(K)$ and $E(K)$. $Q(K)$ and $E(K)$ are subsystems of $T(K)$ for unlimited addition; and the sum, $\Sigma^*R-R \in F$, in $T(K)$ of any subset F of $T(K)$ equals $\Sigma(R_1 \times R_2 \times \dots \times R_n)^3$ where $R_1, \dots, R_n$ are any finite sequence of elements chosen from F .

(c) $O(K)$ is a complete multiplicative system with I as zero, and a subsystem of $B(K)$ for unlimited multiplication.

Proof: We saw earlier (in 2.2) that each of $R(K)$ and $S(K)$ contains the sum as well as the product in $B(K)$ of any subset of itself. So each is a complete lattice and a subsystem of $B(K)$ for unlimited addition and multiplication (by 3.3, (2) and its dual). The units of $R(K)$ are evidently U and I while those of $S(K)$ are U and N . The element $(R' + I)$ satisfies the conditions for the Boolean complement of R in $R(K)$; while, if S is in $S(K)$, S' is also in $S(K)$ and so is the Boolean complement of S in $S(K)$. This completes the proof of (a).

(b) U is the one of $B(K)$ and is in each of $T(K)$, $Q(K)$ and $E(K)$. Hence it must be the one for each of these. Since each

2. That $E(K)$ is a lattice has been proved by Birkhoff; cf. his 'Lattice Theory'—(2) of Bibliography).

3. The Σ here denotes the sum in $B(K)$.

of these sets also contains the product in $B(K)$ of any subset of itself—(by 2.2, (5))—each must be a complete lattice and a subsystem of $B(K)$ for unlimited multiplication (by 3.3, (3)). It is easily verified that N is the zero of $T(K)$, and that I is the zero of $Q(K)$ and $E(K)$.

We shall next show that, for any subset F of $B(K)$, $\Sigma^*R-R \in F$, is the least of the transitive relations which are $\subset$ each R in F . By the definition of Σ^*R , $x(\Sigma^*R)y$ holds if, and only if, a finite sequence of elements $R_1, R_2, \dots, R_n$ of F can be found for which $x(R_1 \times R_2 \times \dots \times R_n)y$ is true. Hence it immediately follows that $x(\Sigma^*R)y$ and $y(\Sigma^*R)z$ imply $x(\Sigma^*R)z$, for any x, y, z from K , so that Σ^*R is transitive. As any R in F can be considered to be a one-element sequence chosen from F , each R in F must be $\subset (\Sigma^*R)$. Finally, if a transitive relation T is $\supset$ each R in F , any relational product of the form $R_1 \times R_2 \times \dots \times R_n$ is $\subset T \times T \times \dots \times T$ to n terms, which is $\subset T$, since T being transitive $T \times T$, and so $T \times T \times \dots \times T$ to n terms, is $\subset T$. Hence Σ^*R is the least transitive relation $\supset$ each R in F .

From the above it follows that, if F is a any subset of $T(K)$, then $\Sigma^*R-R \in F$, is the sum of F in $T(K)$.

If each of a finite sequence of relations $R_1, R_2, \dots, R_n$ is reflexive, so is $R_1 \times R_2 \times \dots \times R_n$; for by repeated use of result (7) of 2.2, $R_1 \subset R_1 \times \dots \times R_n$, and so $I \subset R_1 \subset R_1 \times \dots \times R_n$. And if each of these R 's is symmetric $R_1 \times R_2 \times \dots \times R_n = R_n \times \dots \times R_1$. Hence (Σ^*R) is reflexive if each R is reflexive, and (Σ^*R) is symmetric if each R is symmetric. Thence it follows that $Q(K)$ and $E(K)$ are subsystems of $T(K)$ for unlimited addition. This completes the proof of (b).

(c) This result follows when we note that I is in $O(K)$ and the product in $B(K)$ of any family of ordering relations is an ordering relation (by 2.2, (5)).

Note:—By using the correspondence between equivalence relations on K and partitions of K , it follows that the partitions form a complete lattice when ordered by the relation of one partition being a refinement of the other.

CHAPTER II. CONGRUENCES AND HOMOMORPHISMS

Hereafter we consider sets with associated operations or relations. The relations we consider will be only binary relations; and the operations will always be univocal. (An operation ' θ ' on

the set K is a method of associating with certain subsets of K single elements of K ; the element associated with a subset A is denoted by $\theta(A)$. The operation is univocal if two distinct elements are not associated with the same subset).

§ 5. The congruence relative to an operation.

(5.1) Let θ be any operation defined on the set K . Then:

Def. An equivalence relation, E , on K is said to be a congruence relative to θ if $\theta(c_i) E \theta(d_i)$ holds, whenever $c_i E d_i$ holds for a family of pairs (c_i, d_i) of elements of K , provided $\theta(c_i)$ and $\theta(d_i)$ exist in K .

That is to say: (a) if $\{C_i\}$ is any family of elements of K/E , and if $\theta(c_i)$ exists in K for some choice of elements (c_i) one from each (C_i) , then the coset C relative to E which contains $\theta(c_i)$ is independent of the choice of the elements (c_i) from the cosets (C_i) .

Or, in other words: (b) if an operation θ^* is defined on K/E by writing $C = \theta^*(C_i)$ if, and only if, for some choice of elements (c_i) one from each (C_i) , $\theta(c_i)$ exists in K and lies in C , then θ^* is a univocal operation. (i.e., $\theta^*(C_i)$ is uniquely determined from the family (C_i)).

The operation θ^* so defined on K/E when E is a congruence relative to θ , we shall call the induced operation on K/E . We shall refer to these induced operations again later.

(5.2) The order structure of the subfamily, $K(\theta)$, of $E(K)$ consisting of the congruences relative to θ , when ordered by $\subset$, is given by:

Theorem 2: (i) The family $K(\theta)$ of congruences relative to θ is a complete lattice, and a subsystem of $E(K)$ for unlimited addition. U and I are the one and zero of this lattice.

(ii) If θ satisfies the condition that $\theta(A)$ exists in K for every finite subset A of K , and for such subsets only, then $K(\theta)$ is also a subsystem of $E(K)$ for unlimited addition.⁴

4. This is an improvement on the result of Birkhoff that, under the same assumptions $K(\theta)$ is a lattice and a subsystem of $E(K)$ for finite addition and multiplication (given as Th. 24 in his paper; cf. (1) of Bibliography).

Proof: (i) U and I are in $K(\theta)$. For cUd is true for any elements c, d of K , while cId or $c=d$ for a family of pairs (c_i, d_i) implies $\theta(c_i) = \theta(d_i)$ if these exist, and so $\theta(c_i) I \theta(d_i)$. As $K(\theta)$ is $\subset E(K)$, and U and I are the one and zero of $E(K)$ it follows that they are the one and zero of $K(\theta)$ also.

The product $S = \pi R \cdot R \in F$, in $E(K)$, of any subset F of $K(\theta)$ is also in $K(\theta)$; for if, for a family of pairs (c_i, d_i) , $c_i S d_i$ holds, then for each pair $c_i R d_i$ holds, for each R in F ; so, if $\theta(c_i)$ and $\theta(d_i)$ exist in K , as each R of F is in $K(\theta)$, $\theta(c_i) R \theta(d_i)$ must be true for each R from F . Thence $\theta(c_i) S \theta(d_i)$ follows, so that it is established that S is also in $K(\theta)$. Hence,—by result (3) of 3.3—, $K(\theta)$ is a complete lattice,—with U and I as units—, and a subsystem of $E(K)$ for unlimited multiplication.

(ii) Assuming now that $\theta(A)$ exists in K if, and only if, the subset A of K is finite, we shall show, firstly, that if R, S are in $K(\theta)$, then $\theta(c_i) (R \times S) \theta(d_i)$ is true if (c_i, d_i) is a finite family of pairs for each of which $c_i (R \times S) d_i$ is true. For this last statement implies the existence of a finite family (e_i) of elements of K for which $c_i R e_i$, and $e_i S d_i$ are true. As the set (e_i) is finite $\theta(e_i)$ exists; and, as R, S are in $K(\theta)$, $\theta(c_i) R \theta(e_i)$ and $\theta(e_i) S \theta(d_i)$ follow. Hence our required result, $\theta(c_i) (R \times S) \theta(d_i)$ is established.

If now for a finite family of pairs (c_i, d_i) , $i=1, 2, \dots, n$, $c_i T d_i$ holds, where $T = \Sigma^*(R) \cdot R \in F$, is the sum in $E(K)$ of a subset F of $K(\theta)$, then $c_i R_i d_i$, $i=1, 2, \dots, n$, is true for a set of R_i , each of which is the relational product of a finite sequence of elements of F . If we write $R^* = R_1 \times R_2 \times \dots \times R_n$, it follows that R^* is the relational product of a finite sequence of R 's chosen from F , and so $R^* \subset T$; also, by finite induction on the result of the last paragraph, $\theta(c_i) R^* \theta(d_i)$, and so $\theta(c_i) T \theta(d_i)$, would follow if we proved that $c_i R^* d_i$ is true for each (c_i, d_i) . To prove this we observe that, since each R_i is the relational product of a finite sequence of reflexive relations, they are all reflexive. So, by repeated use of result (7) of 2.2, it follows that $R_i \subset R^*$. Hence $c_i R^* d_i$ follows, for each i , from $c_i R_i d_i$. Thus we have proved that $\theta(c_i) T \theta(d_i)$ follows, if $c_i T d_i$ is true for a finite family of pairs (c_i, d_i) of elements of K . This establishes that T is in $K(\theta)$, since for a non-finite set of pairs (c_i, d_i) , by hypothesis, $\theta(c_i)$ and $\theta(d_i)$ do not exist.

Note: This theorem could be extended, in an obvious manner, to the family $K(\Theta)$ of congruences on K relative to a family Θ , of operations $\{\theta\}$.

§ 6. *Maps and the equivalence relations they define.*

(6.1) A map, (or function), f , from one set K to another K' is a many-one correspondence from K to K' ; we denote the correspondent in K' of an element x of K by $f(x)$. It is assumed further that each element x' of K' occurs as the map $f(x)$ of at least one element x in K .

For any subset A of K , $f(A)$ denotes the set of all $f(x)$, $x \in A$; and for any subset B' of K' , $f^{-1}(B')$ is the set of all x in K for which $f(x)$ is in B' .

Each map, f , from K to K' effects a partitioning of K into non-null, mutually-disjoint subsets; namely, the subsets $f^{-1}(b')$, $b' \in K'$. So to f is associated an equivalence relation $E = E(f)$, related to f by the condition: aEb , for a, b of K if, and only if, $f(a) = f(b)$. We call E "the equivalence on K defined by f ".

(6.2) Among all maps from K to other sets which define the same equivalence E on K there is a specially simple one; namely, the *natural map*, g , from K to K/E which maps each element of K onto the coset relative to E containing it.

A map, f , from K to K' is *reversible* if the equivalence it defines on K is I ; i.e., if, not only does $a=b$ imply $f(a)=f(b)$, but also $f(a)=f(b)$ implies $a=b$. Then $f^{-1}(a')$, for any $a' \in K'$, contains just one element a of K . If we write $a=f^{-1}(a')$, f^{-1} is a map, and also a reversible map, from K' to K . Sets connected by a reversible map may be viewed as obtained one from the other by a re-naming of the elements. Reversible maps are thus, in a sense, trivial.

(6.3) Two maps f, f' ,—one from K to K' and the other from K' to K'' —, may be combined into one map $f''=f'.f$,—from K to K'' under which $f''(x)$, for any x of K , is the same as $f'(f(x))$. We say f'' is composed of f and f' , and that $f'.f$ is a resolution of f'' . (In this resolution f is the first, and f' the second factor). Evidently composition of reversible maps will lead only to reversible maps.

We can resolve any map f , from K to another set K' , which defines on K the equivalence E , in the form $i.g$, where g is the natural map from K to K/E and i is a reversible map from K/E to K' . For, if $i(A)$ is defined, for any A in K/E , to be $f(a)$, where a is any element from A , then $i(A)$ is independent of the choice of a from A ; for if a, b are from A then aEb and so $f(a)=f(b)$ follow. Hence i is a map from K/E to K' ; this map i

is reversible, since $i(A)=i(B)$ implies $f(a)=f(b)$, for some choice of elements a from A and b from B , and so aEb is true for this a and b . Since A, B are the cosets relative to E containing a, b this implies that A and B are identical. That $f=i.g$ is immediately verified.

Hence, neglecting the trivial reversible map factors, the most general type of map from K to other sets is the natural map from K to K/E , E being an arbitrary equivalence on K .

§ 7. *The Homomorphism and the Isomorphism.*

(7.1) A *homomorphism* relative an operation θ , (or relation R), from a set K to another K' is a map, f , from K to K' which carries $\theta, (R)$, defined for K , into a similar operation, $\theta', (relation, R')$, defined for K' ; so that $c=\theta(C)$ in K implies $f(c)=\theta'(f(C))$,—for any subset C and element c of K , (cRd , for any c, d of K , implies $f(c)R'f(d)$).

For example, if the induced operation θ^* on K/E has all the formal properties satisfied by the operation θ on K , where E is a congruence on K relative to a certain operation θ , then the natural map g from K to K/E is evidently a homomorphism from K to K/E . (Note: The need to consider a *similar* operation or relation in K' arises from the fact that in applications we generally consider only homomorphisms between sets which have similar algebraic structure, and relative to those operations or relations which determine their structures).

An *isomorphism* from K to K' relative to θ (or R) is a reversible map i from K to K' , which is such that f and f^{-1} are, respectively, homomorphisms from K to K' relative to θ (or R), and from K' to K relative to θ' (or R').

The composition of two homomorphic or isomorphic maps, evidently, also leads to a map of the same sort.

(7.2) Given a homomorphism, f , from K to K' relative to θ , (or R), which defines on K the equivalence E , we can define such an operation $\bar{\theta}$ (or relation $\bar{R}$), on K/E that in the resolution $f=i.g$ given earlier (in 6.3) g is a homomorphism relative to θ , (or R), and i an isomorphism relative to $\bar{\theta}$, ($\bar{R}$). For we have only to define $\bar{\theta}, (\bar{R})$, by saying: $C=\bar{\theta}(C_i)$, ($C_i \bar{R} D_i$), if, and only if, $i(C)=\theta'(i(C_i))$, ($i(C)R'i(D_i)$), in K' .

Hence treating isomorphism factors as trivial, (since they effect no structural changes), the most general type of homomorphic

map from K to other sets occurs among the natural maps from K to the K/E 's, though the converse, that every natural map is a homomorphic map, is not true, generally. This is due to the fact that the nature of the operation or relation in question imposes further restrictions on the nature of the equivalences on K defined by a homomorphic map. The nature of such restrictions can only be determined when the operation or relation is given specifically. The only general result we can assert in this connection is:

(7.21) "The equivalence E on K defined by a homomorphic map from K to K' relative to an operation, θ , defined for K , must be a congruence relative to θ ". For if $\theta(c_i), \theta(d_i)$ exist in K , where (c_i, d_i) is a family of pairs of the elements from K satisfying $c_i E d_i$, then, since $f(c_i) = f(d_i)$ follows from the last statement, $\theta'(f(c_i)) = \theta'(f(d_i))$ and $f(\theta(c_i)) = \theta'(f(c_i)) = \theta'(f(d_i)) = f(\theta(d_i))$, or $\theta(c_i) E \theta(d_i)$ can be deduced. Hence E is a congruence relative to θ . The further restrictions that may also be imposed on the equivalence on K defined by a homomorphic map relative to an operation, can be discussed only if we know the precise nature of the operation.

(7.3) For partially ordered sets, since there is a basic ordering relation which determines the structure, we define a homomorphism between ordered sets to be, in the first instance, relative to the ordering relation; it may also be relative to some specified set of operations defined in terms of the ordering relation. An isomorphism between ordered sets is just a reversible map which carries the ordering relations of the two sets one into the other. An important example of such an isomorphism is considered in the coming section.

§8. The isomorphism between the principal α -ideal

$P_\alpha(E)$ of $Q(K)$ and $Q(K/E)$.

Theorem 3: (i) Let E be any equivalence relation on a set K . Then there is an isomorphism $j, = j(E)$, from the principal α -ideal $P_\alpha(E)$ of the lattice $Q(K)$ to the lattice $Q(K/E)$; under this isomorphism, equivalence relations on K (in $P_\alpha(E)$) correspond to equivalence relations on K/E .

5. The family of elements of an ordered set which are $>$ a fixed element c is called the "principal α -ideal generated by c ", and denoted by $P_\alpha(c)$.

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(ii) If E is a congruence on K relative to an operation θ , and if θ^* is the induced operation on K/E , the isomorphism correlates congruences on K relative to θ with the congruences on K/E relative to θ^* .

Proof: Given an element R of $Q(K)$ which is $\supset E$, we define a relation $R^* = j(R)$, by: AR^*B , for A, B of K/E , if, and only if, aRb for some a from A and some b from B . This implies, (and evidently is implied by), $a'Rb'$, whatever a', b' be from A, B ; for aRb , $a' \in A$, $b' \in B$, imply, $a'Ea$, aRb and bEb' ; and, as $E \subset R$, it follows that $a'Ra$, bRb' . As R is transitive $a'Rb'$ follows.

With such a definition of $j(R)$, $j(R)$ is seen to be reflexive and transitive, since R is reflexive and transitive. (aRa , for all a in K , implies AR^*A , for all A in K/E ; while, AR^*B , BR^*C , for A, B, C from K/E , imply aRb , bRc if a, b, c are in A, B, C ; thence aRc and AR^*C follow). Also, given any R^* in $Q(K/E)$, if a relation R on K is defined by: aRb if, and only if, AR^*B , for the cosets A, B relative to E containing a, b , R is seen to be a transitive relation $\supset E$. (For aRb , bRc imply AR^*B , BR^*C and so AR^*C and aRc ; while aEb implies $A=B$, and as R^* is reflexive, $AR^*B \cdot Eo aRb$ follows). Hence R is in $P_\alpha(E)$ and evidently $j(R) = R^*$. So j maps $P_\alpha(E)$ of $Q(K)$ on $Q(K/E)$.

This is a reversible map; for if $R^* = j(R) = j(S) = S^*$, where R, S , are in $Q(K)$, aRb implies AR^*B or AS^*B and so aSb ; and similarly we can prove aSb implies aRb . Thus $R=S$ if $j(R) = j(S)$.

Finally j is an isomorphism. For if $R \subset S$, $Aj(R)B$ implies aRb , if $a \in A$ and $b \in B$; and aRb implies aSb and so $Aj(S)B$. So $j(R) \subset j(S)$ if $R \subset S$. While if $j(R) \subset j(S)$, aRb implies $Aj(R)B$; and $Aj(R)B$ implies $Aj(S)b$ and so aSb . Hence $j(R) \subset j(S)$ also implies $R \subset S$.

It is evident that $j(R^{-1}) = (j(R))^{-1}$ for any R in $P_\alpha(E)$ of $Q(K)$. Hence R is symmetric—and so an equivalence relation—if and only if $j(R)$ is symmetric—and an equivalence relation. This completes the proof of part (i).

(ii) Let us now assume that E is a congruence on K relative to an operation θ , and θ^* is the induced operation on K/E . If R is a congruence relative to θ then R , and so $R^* = j(R)$, is an equivalence relation. To prove that R^* is also a congruence on K/E relative to θ^* , suppose $C = \theta^*(C_i)$, $D = \theta^*(D_i)$, exist in K/E , where (C_i, D_i) is a family of pairs from K/E for which $C_i R^* D_i$ is true. Then, by the definition of θ^* , elements c_i one from each

C_i , and elements d_i one from each D_i can be chosen so that $\theta(c_i)$ exists in K and lies in C , while $\theta(d_i)$ also exists and lies in D . And $C_i R^* D_i$ implies, $c_i R d_i$, for these (c_i, d_i) . As R is a congruence relative to θ , $\theta(c_i) R \theta(d_i)$, and hence CR^*D follow. This proves that R^* is a congruence relative to θ^* .

If, conversely, R^* is a congruence relative to θ^* , as before, R must be an equivalence relation. And if $\theta(c_i), \theta(d_i)$ exist in K , where (c_i, d_i) is a set of pairs from K for which $c_i R d_i$ holds, then CR^*D follows for the cosets C, D relative to E containing $\theta(c_i)$ and $\theta(d_i)$, since $C_i R^* D_i$, and $C = \theta^*(C_i), D = \theta^*(D_i)$ are true, where R^* is a congruence relative to θ^* . Hence $\theta(c_i) R \theta(d_i)$ follows, proving that R is a congruence relative to θ .

In the next section we relate the sets K/R and $(K/E)/R^*$, when $R^* = j(R)$, and R, R^* are equivalence relations.

§9. The reversible map between K/R and $(K/E)/R^*$.

As in the last section E is an equivalence relation on K and J the isomorphism between $P_a(E)$ of $Q(K)$ and $Q(K/E)$.

Theorem 4: (i) If R, R^* are corresponding equivalence relations on K and K/E under the isomorphism j , and if we denote by g, f and f^* the natural maps from K to K/R , from K to K/E , and from K/E to $(K/E)/R^*$ respectively, there is a reversible map i from K/R to $(K/E)/R^*$ such that $f^* \cdot f = i \cdot g$.

(ii) If further E is a congruence on K relative to an operation θ , and R, R^* are congruences relative to θ and θ^* —(as in part (ii) of Th. 3)—and if we denote by $\bar{\theta}, \bar{\theta}^*$ respectively the induced operations on K/R and $(K/E)/R^*$, then the map i defined above is an isomorphism relative to $\bar{\theta}$, carrying $\bar{\theta}$ into $\bar{\theta}^*$.

Proof: (i) If R, R^* are corresponding equivalence relations on K and on K/E , from the definition of $R^* = j(R)$, it follows that aRb if and only if AR^*B , where a, b are elements of K and A, B are the cosets relative to E containing them. Hence each coset $\bar{A}$ of K relative to R is the set-union of a family, A^* , of cosets of K relative to E ; and the family A^* is a coset of K/E relative to R^* . This correlation between an element $\bar{A}$ of K/R and an element A^* of $(K/E)/R^*$ is evidently (1-1) reversible. Hence if we define $i(\bar{A}) = A^*$, i is a reversible map from K/R to $(K/E)/R^*$. It is evident that, for any element a of K , $f^*(f(a)) = f^*(\bar{A}) = A^* = i(\bar{A}) = i(g(a))$. Hence $f^* \cdot f = i \cdot g$.

(ii) When R, R^*, E are congruences relative to θ, θ^*, θ , the definitions of $\theta^*, \bar{\theta}$, and $\bar{\theta}^*$ imply the logical equivalence of the following propositions where $c, (c_i)$ are arbitrary elements of K : $g(c) = \bar{\theta}(g(c_i))$ in K/R ; $c = \theta(c_i)$ in K ; $f(c) = \theta^*(f(c_i))$ in K/E ; and $f^*(f(c)) = \bar{\theta}^*(f^*(f(c_i)))$ in $(K/E)/R^*$. Hence $g(c) = \bar{\theta}(g(c_i))$ if, and only if, $i \cdot g(c) = f^* \cdot f(c) = \bar{\theta}^*(f^* \cdot f(c_i)) = \bar{\theta}^*(i \cdot g(c_i))$, so that i is an isomorphism from K/R to $(K/E)/R^*$, carrying $\bar{\theta}$ into $\bar{\theta}^*$.

§10. The example of the Group Homomorphism.

The theory of congruences and homomorphisms for Groups—relative to the group-operation—is specially simple.

The simplicity arises from the fact that any congruence, E , on a group G is completely specified by a single one of the subsets into which G is partitioned by E ; namely, the normal subgroup $N = N(E)$, of elements of G congruent under E to the identity element, 1 , of the group. The relation between E and N is: aEb , for a, b of G , if and only if $a \cdot b^{-1} \in N$. Since N is also defined completely by E , there is a (1-1) reversible correspondence between the congruences on G and the normal subgroups of G . This obviously carries the ordering relation among the congruences into the set-inclusion relation among the normal-subgroups, and vice-versa. Hence ordered by set-inclusion the normal subgroups must form a complete lattice isomorphic to the lattice of congruences. (It is further known that this lattice is also modular; cf., Birkhoff's book on 'Lattice Theory').

The quotient group G/N , as it is usually defined, is seen to be the same set as G/E , having the induced operation defined on G/E as the group-operation. So, it follows, in particular, that any congruence E on the group G can be identified with one of those defined by homomorphisms from G to other groups,—for instance the (natural) homomorphic map from G to $G/E = G/N$.

Finally the above correlation, between any congruence E and a certain normal subgroup N , under which the induced operation on $G/E = G/N$ is, the group operation of G/N , enables us to deduce, from Theorem 4, the following:

If G is any group, N any normal subgroup of G , and $\bar{G} = G/N$, then (a) if $\bar{N}$ is a normal subgroup of $\bar{G}$, the set union N' of the residue classes of G modulo N which are in $\bar{N}$ is a normal sub-

group of G , and G/N' is isomorphic to $\bar{G}/\bar{N}$;⁶ and (b) if N' is a normal subgroup of G containing N , it is the set-union of a set $\bar{N}$ of complete residue classes of G modulo N ; this set $\bar{N}$ is a normal subgroup of $\bar{G}$, such that G/N' is isomorphic to $\bar{G}/\bar{N}$.

The results (a) and (b) give respectively the method for the composition and resolution of group homomorphisms. Similar results I propose to deduce for additive-, and lattice-homomorphisms in a future paper.

BIBLIOGRAPHY

1. G. Birkhoff: "On the structure of Abstract Algebras", *Proc., Camb., Phil., Soc.*, Vol. 31, 1935, (433-54).
2. G. Birkhoff: "Lattice Theory", *Am., Math., Soc., Colloq. Publ.*, Vol. XXV.
3. P. Dubreil and M.-L. Dubreil Jacotin: "Theorie Algebrique des relations d'equivalence", *Jour. de Math.*, 18, 1939, (63-96).
4. H. M. MacNeille: "Partially Ordered Sets", *Trans. Am. Math. Soc.*, Vol. 42, 1937, (416-60).
5. O. Ore: "Theory of Equivalence relations", *Duke Math. Jour.*, Vol. 9, 1942, (573-672); this journal is not available for reference in the Madras University Library.

6. This is the same as the Second Isomorphism Theorem given in Van der Waerden's "Moderne Algebra" (Teil 1, Ch. VI, §40, p. 136).

A STUDY OF THE SUCCESSION OF TEETH
IN ELASMOBRANCHS

By

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INTRODUCTION

It has been held by several competent workers such as Kerr, Norman, Gudger, etc., that Elasmobranchs replace their teeth constantly—a kind of dentition often described as lyodont, i.e., several sets of teeth functioning in succession. Recently a series of short papers have been published by Cawston (1938-42) refuting the successional theory of teeth, by the rolling outwards of the tooth-bearing area in Elasmobranchs and suggesting replacement by vertical succession. At the suggestion of Prof. R. Gopala Aiyar a study of the dentition of some of the local Elasmobranchs with the object of elucidating the problem was started and the following is a brief statement of the result.

Two different methods have been employed in these investigations, (1) a comparative study of the dentition of the adults and embryos and (2) laboratory experiments involving the keeping alive of young Elasmobranchs with artificially damaged dental armature.

Interest in the dentition of Elasmobranchs goes back to the 18th century. Very little attention, however, seems to have been paid to the actual method of replacement of teeth. One of the earliest papers in this connection is by William André (1784) where the author attempts to prove from data based on accidental circumstances that the teeth of cartilaginous fishes are perpetually renewed.

Kerr (1919) in his "Text Book of Embryology" says "The portion of the skin—ectoderm with its strong and fibrous underlying dermis—to which the tooth-bases are attached is gradually, by a process of differential growth, caused to shift its position in an outward direction over the edge of the jaw which supports it."

Bridge (1922) writes:—"As the teeth in use become lost they are successively replaced by the inner rows, which, with the mucous membrane in which they are embedded, move forwards to the edge of the jaw, where they become erect and functional." What is apparently meant here is, 'forwards' in regard to the teeth at the front end of the jaw and outwards in regard to the teeth on the sides of the jaw.

Norman (1936) in "History of Fishes" says, "As the functional teeth are lost those of the next series move upwards, and the whole phalanx is thus constantly marching onwards throughout life, a row of teeth doing duty for a time, only to be duly cast off in its turn and replaced by its successor."

Gudger (1937) in his paper on the 'Abnormal dentition of Sharks' cites more instances where defective sets of teeth were formed due to the piercing of the tooth-germs by the stings of sting rays. He agrees with William André in the matter of the mode of succession of teeth in Elasmobranchs.

Cawston (1938-42), who has published a series of short articles on the succession of teeth in Elasmobranchs, questions the accepted view of the mode of succession. He believes that there is no lateral revolving of the area of tooth development and that among fishes if at all there is succession it is vertical.

In a very recent paper, of which a summary alone has been available to me, Breder (1942) states that *Carcharias littoralis* sheds the teeth and that they are replaced in regular succession.

It gives me great pleasure to take this opportunity to thank Prof. R. Gopala Aiyer, for his constant help and very valuable criticism. I also wish to thank the University of Madras for awarding me a Research Studentship.

MATERIAL AND METHODS

The period of collection extended from March 1942 to April 1943. Jaws of adult specimens of *Hemigaleus balfouri*, *Carcharinus melanopterus*, *Chiloscyllium griseum*, *Stegostoma tigrinum*, *Scoliodon sorrakowah*, *Rhinobatus granulatus*, *Rhynchobatus djeddensis*, *Pristis cuspidatus*, *Trygon uarnak*, *Astrape dipterygia*, *Narcine timlei*, *N. brunnea* and *Hypolophus sephen* were collected. Small portions of the adult jaws were fixed either in Sansom's fluid or Pereney's fluid and before sectioning the material was thoroughly decalcified. Initial difficulty encountered in getting

good micro-preparations of the entire series of teeth in situ was slowly overcome. Of the various methods of decalcification, Nitric acid and alcohol was found most suitable. Phloroglucin-nitric acid method was not very successful. Transverse sections of the jaws were taken at 8-12 μ thickness. Sections of the jaws of embryos of some forms, which were also collected, were also taken.

Other fixatives used were Formalin 5%, Zenker formol, Bouin's fluid and Picro-sulphuric. Delafield's haematoxylin gave better results than Heidenhain's iron-haematoxylin. Mann's methyl-blue-eosin and Mallory's triple stain were also used.

ARRANGEMENT OF TEETH IN EMBRYOS AND ADULTS

A comparative study of the arrangement of teeth in the adults and embryos of *H. balfouri*, *C. griseum*, *R. granulatus*, *T. uarnak* and *N. brunnea* has been made. In all these cases the arrangement of teeth is the same in the adult as well as in the embryo. Sections of the jaws of adults and embryos reveal that teeth are always found to develop one behind the other in regular succession (Photograph 1). The outermost will be best developed, the innermost the least developed and the middle ones of intermediate stages. Thus a transverse section of the jaw of either an embryo or an adult shows the various stages in the development of teeth.

In the embryos fully formed teeth are of the same size with pointed cusps (wherever pointed cusps are present). Whereas examination of the adult jaws by sectioning or under the binocular microscope shows that the front row of teeth are all blunt. Sections of the jaws of different adult specimens examined revealed the same wear and tear. This wearing down of teeth is well seen in the jaws of adult *S. tigrinum* (Photomicro. 2). It is observed that the outermost one or two rows of teeth (which have been pushed beyond the edge of the jaw) are very blunt, being devoid of pointed cusps. Figures 1, 2, and 3 show a normal tooth and two stages in the wearing down of tooth respectively. Careful examination of sections of these teeth shows that the enamel coating gets worn out gradually resulting in the complete disappearance on the outermost one or two rows of teeth. This lends additional proof to the fact that these are only worn out teeth which are being gradually pushed outwards. That the wear is gradual could be seen in that the outermost one or two rows are very blunt, whereas

those immediately behind them have short blunt cusps and those unworn teeth have long pointed cusps.

In the early embryonic condition teeth are all covered over by the oral mucous membrane (Photomicro. 3). But as the embryo grows the dental lamina also grows and in the adults it extends beyond the edge of the jaw (Photomicro. 4). The same has been noted in other forms also.

Examination of the jaws of several adult specimens did not reveal any with an abnormal dentition brought about by injuries such as those mentioned by André (1784) and Gudger (1937).

SUCCESSION OF TEETH

It is believed that nearly all fishes are polyphyodont and it is supposed that the old teeth as they become worn out or fall down, are constantly replaced by new teeth.

In Elasmobranchs the teeth are arranged in rows parallel to the axis of the jaw, the functional teeth being exposed at the outer edge of the jaws, where they are usually erect. There is no firm attachment or ankylosis of the teeth to the jaws in these fishes, and they are simply embedded in the gums.

Until recently it was supposed that Elasmobranchs perpetually renew their teeth by the rolling outwards of the tooth-bearing area. This mode of replacement has been very aptly compared to the fall of the front row of soldiers being replaced by those from behind. Similarly as the front row of functional teeth are lost or worn out the next row behind it marches outwards and takes its place. Even though there were no concrete proofs for assuming this type of replacement earlier workers have, on the basis of the arrangement of teeth in Elasmobranchs, supposed that the fallen teeth of the front row get replaced from behind. In all Elasmobranchs usually more than one set of teeth are functional and on an examination of the jaw either by dissection or through sections one could find rows of teeth arranged in regular succession one behind the other, the rows of teeth belonging to various stages of development. The unexposed teeth are hidden beneath the projecting fold of the oral mucous membrane.

The widely accepted successional theory came for criticism at the hands of Cawston who in a series of articles has tried to throw doubt on the existing view.

Cawston (1942) says "Examination of sharks at various stages of growth failed to reveal evidence of shedding except for an occasional exposed tooth, whilst minute teeth at the foremost position of the teeth of some sharks rendered very remote the possibility of their being replaced by the stouter hinder ones. Nor did it seem likely that the hindermost poorly developed teeth would ever move forward to replace those stout but seldom used teeth which are usually covered by gum unless this is torn away."

According to him even if there is a remote possibility of any replacement it could not be by the forward progression of the tooth-bearing area but can only be effected by means of vertical succession. "As such replacement by forward progression is quite impossible in some sharks, such as the Devil Ray, it seemed a mistake to conceive some means of replacement other than by vertical succession and, for want of experimental proof, investigators were made to discover whether any other animal than sharks could replace their teeth in any way but by vertical succession." In support of his view that there is no loss of teeth due to wear and tear he remarks that there are only few evidences of lost teeth reported among fishermen in the shark industry. "Scientists have surmised that sharks are constantly shedding their teeth and that these are replaced by teeth from the hinder rows. It has not been possible to prove this theory by experimental tests, and there are remarkably few occasions where evidence of lost teeth has been reported among fishermen engaged in the shark industry."

He examined embryonic and adult rays and skates where he says forward progression is very unlikely due to the closely attached nature of the teeth to the underlying cartilage and also that the flattening of the front teeth is constant throughout life unrelated to wear and tear. X-ray examinations and dissections of various jaws made by him demonstrated, according to him, vertical succession and he says that the only position in which it seemed possible to apply the successional theory was in that of the grooved teeth of the front fanged snakes, where serial development also lent support to the belief that the lost fang might be replaced. From these observations he concludes as follows. "These studies indicate that replacement of teeth under natural conditions occur only by vertical succession, and that the successional theory, involving a constant forward movement of the tooth-bearing area, is founded on assumptions for which there is no justification."

Regarding William André's observations Cawston (1942) regards it as convincing proof that the abnormal teeth there deg-

cribed were due not to any progression of the gum after the injury but to disturbances of the dental germ at the injured site.

Breder (1942) as already mentioned is the only author who as a result of aquarium experiments, has observed that there is shedding of teeth in Elasmobranchs and that they are replaced in regular succession.

A word or two about the mode of replacement of teeth in other vertebrates may not be out of place here. Warren (1902) has shown that the mode of replacement of teeth in Cyclostomes is by vertical succession. In Teleostomi as said by Bridge (1922) the succession is not as regular as in Elasmobranchs. New teeth are formed between or at the base of the old teeth. In the case of socketed teeth the succession is usually vertical the new teeth being formed at the sides of the old ones; and by the absorption of the bases of the latter, the former comes to lie directly below them, and eventually they occupy the same sockets. In Amphibia, it is said, that the general arrangement for replacement of teeth is similar to those of Elasmobranchs but an advance beyond the Elasmobranch condition is found here in that the functional teeth are firmly fused to the jaw and when it is lost either by being broken off or by a process of natural shedding accompanied by resorption, the next replacement tooth in the series moves up to take its place. In reptiles again the general arrangement is similar but it has been observed already that amongst certain lizards modifications occur which are of importance as foreshadowing arrangements which occur in mammals. Thus Kerr remarks that in *Iguana* the first generation of teeth never become functional. Again the replacement mechanism may become reduced in the anterior part of the series as in *Agama* and he says that the ordinary replacement mechanism is no longer functional, in Chameleon, except at the extreme hind end of the jaw where alone new teeth are produced, each one lying to the inner side of the functional tooth. As the successional tooth develops it causes absorption of the inner wall of the functional tooth, and gradually comes to lie within the base of the latter. Finally the old tooth is shed and its successor remains in its place. In mammals it is well known that the mode of replacement is by vertical succession.

A comparative study of the jaws of embryos and adults of various Elasmobranchs as already stated shows that the more erect anteriormost teeth in the adults are smaller and imperfect showing definite signs of wear and tear. On the other hand all the fully formed teeth of the embryo are of the same size. In all

the adult rays examined by me I was able to note that the central pointed elevation of exposed teeth shows clear signs of wear and tear whereas those that are arranged behind, which do not come into active play, have their cusps pointed and sharp. In the case of *S. tigrinum*, as already pointed out, the wear and tear is very well pronounced.

Till now actual empirical proof was lacking and with a view to prove that there is replacement of teeth and that too not by vertical succession but by the rolling forward of the tooth-bearing area experiments were carried out on live specimens kept in aquarium tanks.

Specimens of *T. uarnak*, and *N. timlei* were kept in the laboratory aquarium tanks. Extreme difficulty was encountered in keeping these for a sufficiently long time. They were fed on small *Hippa*. First a specimen of *N. timlei* with a few front rows of teeth removed was kept in the aquarium tank. This died after five days, probably due to the shock received during the removal of teeth. A section of the jaw revealed that there was partial replacement. The same experiment was repeated on *T. uarnak*. Teeth were removed as shown in Fig. 4. AA' indicates the line through which the cut was made. It survived for 12 days and an examination of a section of the jaw, passing through the region where the teeth had been removed, revealed that the complete set has been replaced and there was absolutely no sign of any abnormality.

Two other specimens of *T. uarnak* were obtained. Here the teeth were removed in a different manner. This is shown in Fig. 4. From the centre of the lower jaw a piece of the tooth-bearing area about 1 mm. in width was cut off thus removing the complete set of exposed vertical rows. This method was adopted so that sections passing just outside the region from where teeth have been removed can be compared with sections of the region from where teeth have not been removed and thus to ensure that the replacement is complete and that the injured portion regains the normal condition. These specimens were left in the aquarium tank and were constantly examined. One of them was killed on the 8th day. Sections were taken of the entire jaw through the uninjured and the injured regions.

A section (Photomicro. 5) passing through that portion of the jaw indicated by the line XY in Fig. 4 where the dental armature was partly removed, on comparing with a section (Photomicro. 6)

passing through X' Y' where the teeth have not been injured, shows that the gap at the tip of the jaw in the former case has only been partially bridged over. This indicates that the replacement is not complete yet, but the presence of an exposed row of teeth shows that there has been partial replacement. The other specimen was kept in the aquarium tank for 24 days and on the 24th day it was taken out and on examination with a hand lens showed that the replacement is complete. The jaw was removed and fixed. Sections passing through similar regions as indicated by XY and X' Y' in Fig. 4 were compared (Photomicrographs 7 and 8). Sections passing through that region of the jaw where teeth have been removed and made to replace are identical with those sections passing through the region where the teeth were not disturbed.

These experiments conclusively prove that there is replacement if teeth are lost or worn out. According to Cawston's view the replacement must be by vertical succession. If it is so new tooth-germs must be seen below the removed teeth. But there is absolutely no sign of any tooth-germ developing at the site where the teeth have been removed; on the other hand new tooth-germs are seen developing as usual one behind the other in the dental groove covered over by the oral mucous membrane. In these jaws there is absolutely no abnormality like crowding of teeth at the place where teeth have been removed nor is there any proliferation of cells at the site of injury. Activity is confined to the dental groove in which new tooth-germs are being developed in regular succession, one behind the other. By this process of replacement and formation of new teeth a practically constant number of rows of teeth in an animal is maintained and this phenomenon explains the constancy in the number of rows in all the adults of a species.

The same experiment was tried on a specimen of *C. griseum* and a similar result was obtained. Due to the great difficulty in obtaining the necessary number of healthy live specimens and also due to difficulties experienced in keeping them alive in the aquarium tanks for a sufficient length of time I could not extend the experiments on to other forms.

It is not difficult to conceive the rolling outwards of the tooth-bearing area in Elasmobranchs even without these experiments. In the embryonic condition the whole set of teeth are covered over by a fold of the oral mucous membrane. It is evident that unless these teeth move outwards it is impossible for them to occupy the

edge of the jaw. Kerr (1919) remarks that the portion of the skin—ectoderm with its strong and underlying dermis—to which the tooth-bases are attached is gradually, by a process of differential growth, caused to shift its position in an outward direction over the edge of the jaw which supports it. This is brought about by the skin undergoing a continual slow process of absorption or atrophy along the outer margin of the jaw, near the bottom of a deep groove. In the adults of most rays and skates the dental lamina curves round the tip of the jaw, a process brought about by the moving outwards of the tooth-bearing area. Moreover an examination of the jaw of an adult shark (Photograph 9) shows that the teeth at the edge of the jaw are erect whereas the replacing teeth behind are oblique or slanting.

The facts adduced and the experimental proof given prove beyond doubt that the theory of replacement by rolling outwards is the correct one for Elasmobranchs. It may therefore be said with William André that "a part of the inhabitants of the great deep, retain in the article of teeth, a perpetual juvenility being utter strangers to an edentulous old age."

SUMMARY

1. A comparative study of the arrangement of teeth in the adults and in some cases of the embryos of Elasmobranchs has been made.
2. It is observed that there is wear and tear of teeth.
3. The method of replacement of teeth artificially removed is described.
4. It is observed that replacement is not by vertical succession but by the rolling outwards of the tooth-bearing area.



BIBLIOGRAPHY

- André, W.* 1784 "A description of the teeth of the *Anarrhichas lupus*, Linnaei and those of *Chaetodon margicans* of the same author," Phil. Trans Roy. Soc., Lond., Ser. B. Vol. 74.
- Breder, C.M. Jr.* 1942 "The shedding of teeth by *Carcharias litto-ralis* (Michell)." Copeia, Vol. 1.
- Bridge, T.W. • 1922 "Cambridge Natural History." London. Vol. 7, Fishes and Ascidians, etc.
- Burckhardt, R. 1906 "Handbuch der vergleichenden und Experimentellen Entwicklungslehre der Wirbel-tiere." Jena., Bd. 2.
- Cawston, F.G. 1938 "Succession of teeth in Sharks, Selachii." South. Afric. Jour. Sci., 35.
- _____• 1939 "A consideration of the possible succession of teeth in fishes and reptiles," Dent. Rec., 59. No. 10.
- _____ 1939 "Sharks and their teeth." Boys' Mag., Vol. 53, No. 12.
- _____• 1940 "A consideration of the possible succession of teeth in Rays." Dent. Rec., Vol. 60, No. 3.
- _____• 1940 "A study of snakes in relation to the succes-sional theory." Ibid. Vol. 60. No. 8.
- _____ 1940 "The dentition of fishes and reptiles with special reference to the replacement of teeth." Ind. Jour. Vet. Sci. and Animal Hus. Vol. 3. Pt. 10.
- _____• 1940 "A consideration of the alleged succession of teeth by revolving of the tooth-bearing area." S. Afric. Dent. Jour., Vol. 14. No. 1.
- _____ 1940 "U Bespreking van die tande van *Carcharias taurus*." Tydskrif. Wetenskap en Kuns. 1 (2).
- _____• 1940 "A consideration of the replacement of teeth in Sharks and fangs in Snakes." Dent. Rec. Vol. 60. No. 11.
- _____ 1941 "A consideration of the teeth of embryonic and immature Skates and Rays in relation to the successional theory of teeth." S. Afric. Dent. Jour., Vol. 15. No. 4.

- Cawston, F.G. 1941 "A consideration of the recurved teeth and reserve fangs of Snakes." Dent. Rec. 61 (4).
- _____ 1941 "Die Tande-Draende Streek van die Tand-rulus van Selachii, Batoidei in Ander visse." Tydskrif. Wetenskap en Kuns. Bd. 2.
- _____• 1941 "A note on the development and survival of teeth, especially Selachian." Dent. Rec. 61. (10).
- _____ 1942 "A consideration of the successional theory of teeth." Current Science, Vol. 8 (11).
- Gudger, E.W.* 1933 "Abnormal dentition in Rays, Batoidei." Jour. Elisha Mitchell. Sci. Soc. Chapen Hill. N.C. Vol. XLIX.
- _____ 1937 "Abnormal dentition in Sharks." Bull. Amer. Mus. Nat. His., Vol. 73.
- Kerr, J.G. 1919 "Text book of embryology." London. Vol. 2.
- Norman, J.R. 1936 "A History of Fishes." London.
- Warren, E. 1902 "Teeth of Petromyzon and Myxine." Q.J.M.S. Vol. 45.

Note.—References marked with an asterisk have not been referred to in the original.

EXPLANATION OF FIGURES AND PHOTOGRAPHS

Plate I. Figures.

1. A normal unworn tooth in the lower jaw of *S. tigrinum*.
2. A partly worn out tooth in the same jaw of *S. tigrinum*.
3. A much worn out tooth from outermost row in the lower jaw of *S. tigrinum*.
4. A diagrammatic representation of the lower jaw of *T. uarnak* showing the regions from which the dental armature was removed and also the planes through which sections were taken. All teeth in front of the line A A' were removed in the first experiment, in the second, teeth were removed from the cross-hatched region. Lines XY and X' Y' indicate planes through which sections were taken.

Plate II. Photographs.

Photograph:

1. A transverse section of the lower jaw of *H. sephen* showing the arrangement of teeth.

Photomicrographs :

2. A transverse section of the lower jaw of *S. tigrinum* showing the gradual wearing out of teeth. Note absence of pointed cusps from teeth beyond front edge of the jaw.
3. A transverse section of the lower jaw of an embryo of *N. brunnea*. Note the entire set of teeth covered over by the oral mucous membrane.
4. A transverse section of the lower jaw of an adult *N. brunnea*. Note dental lamina curving round the edge of the jaw and also the gradual wearing out of teeth. Note absence of pointed cusps from teeth beyond front edge of the jaw.
5. A transverse section passing through XY of Fig. 4 showing gap, only partially bridged over by replacement of teeth. (Second specimen of *T. uarnak* mentioned in the text).
6. A transverse section passing through X' Y' of Fig. 4 showing the normal condition. (Same specimen as above).
7. A transverse section passing through XY of Fig. 4 showing complete replacement of teeth. (Third specimen of *T. uarnak*).
8. A transverse section passing through X' Y' of Fig. 4 to show the normal condition. (Third specimen of *T. uarnak*).

Photograph :

9. A jaw of an adult *C. melanopterus* showing the arrangement of teeth. A portion of the oral mucous membrane (indicated by a circle) has been removed to show the slanting replacing teeth.

KEY TO LETTERING

ga.		Gap partially bridged over by the replacement of teeth.
m.c.		Meckel's cartilage.
o.m.m.		Oral mucous membrane.
r.t.		Replacing teeth.
t.		Teeth.
w.t.		Worn out teeth.

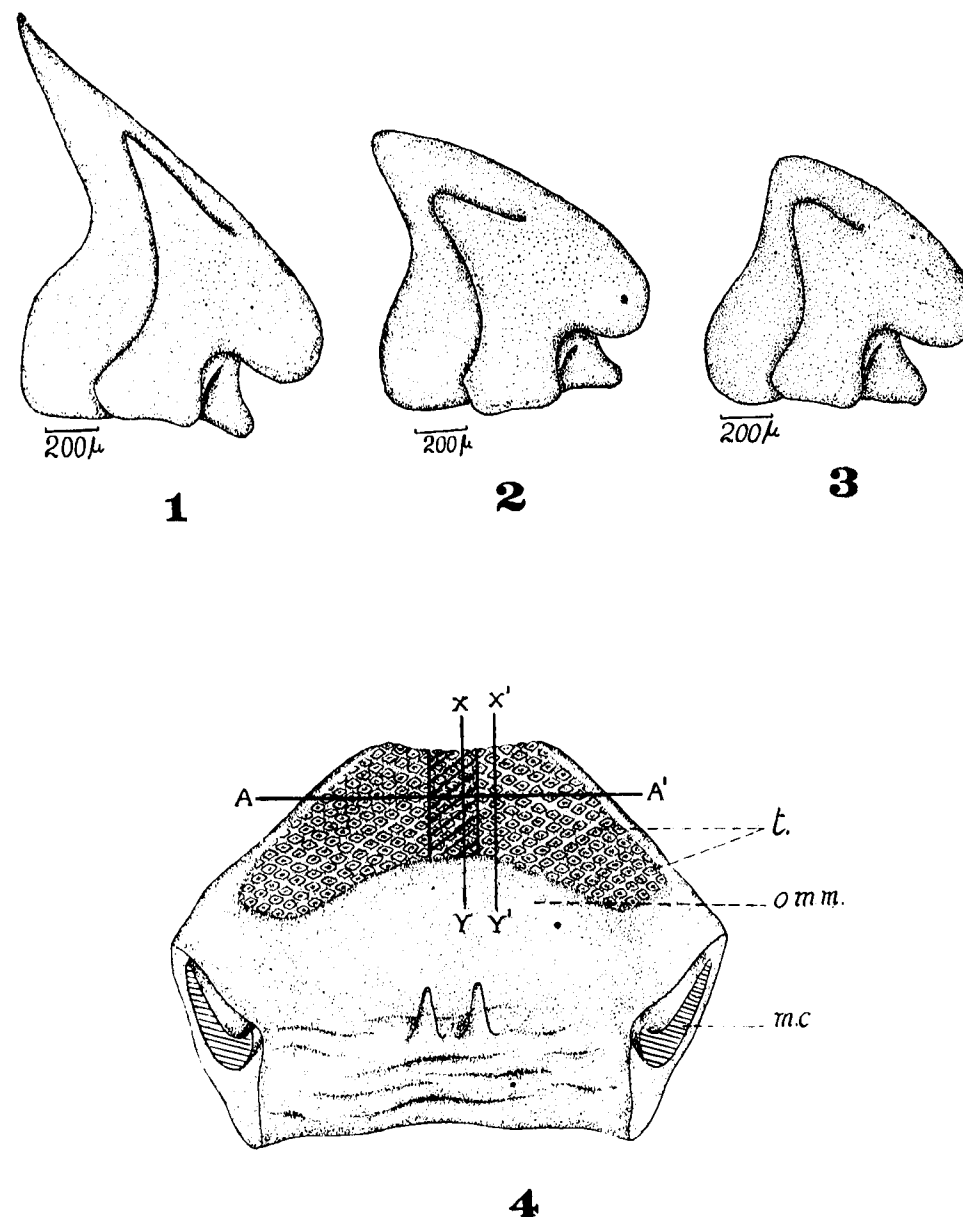
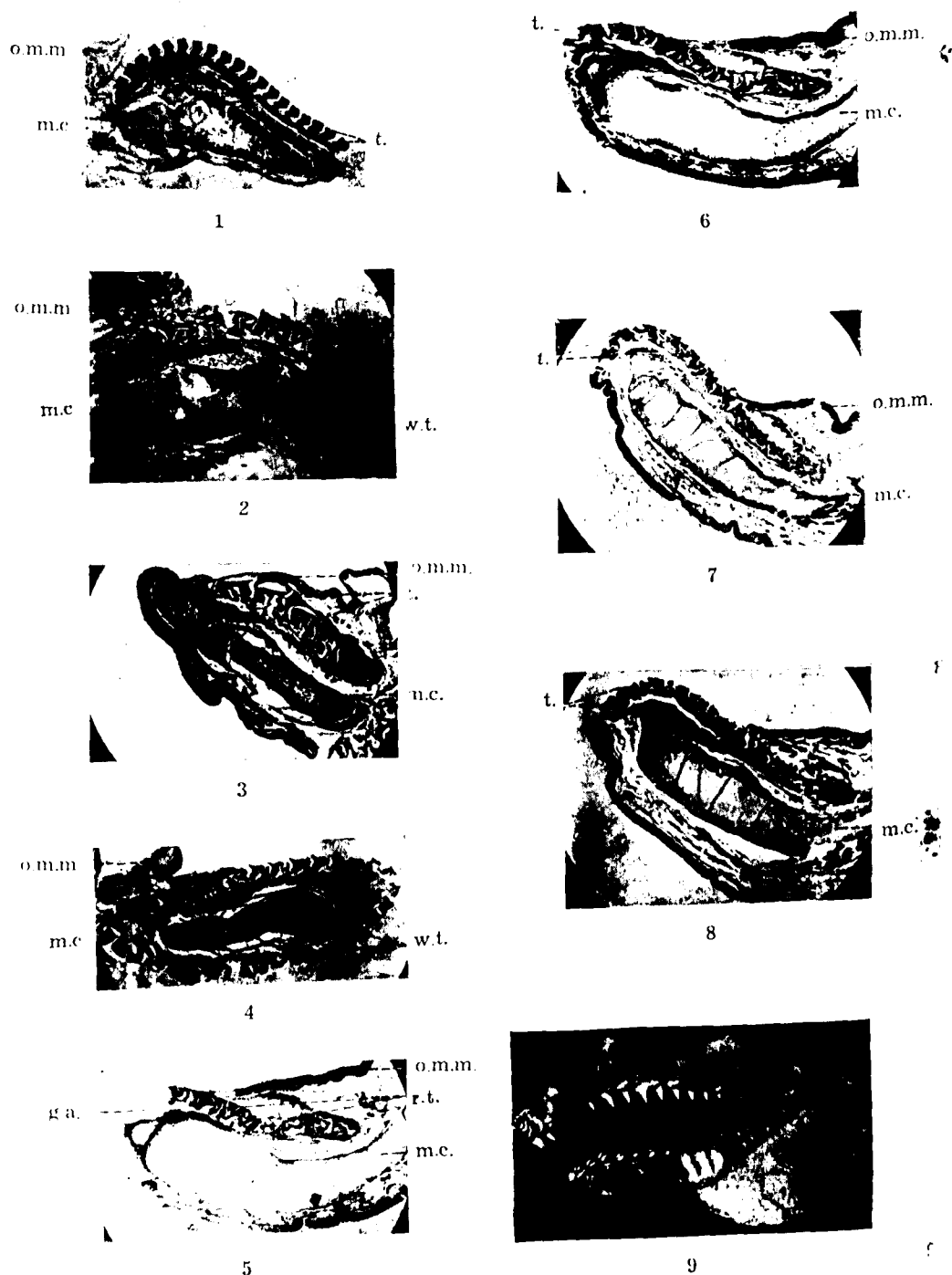


PLATE II



A SYSTEMATIC ACCOUNT OF SOME MARINE
MYXOPHYCEAE OF THE SOUTH INDIAN COAST*

By

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Very little work has been done so far on the marine Myxophyceae of the Indian Coast.

Martens (1866) recorded three species of blue-green algae from near Ceylon, viz., (1) *Oscillatoria subsalsa* Ag., (2) *Microcoleus chthonoplastes* Thuret and (3) *Lyngbya majuscula* Harvey. Murray, in 1887, published an account of some Ceylon algae, in which he has recorded, besides other algae, six species of blue-green algae. Of these only two were marine, viz., *Lyngbya majuscula* Harvey and *Trichodesmium erythraeum* Ehrenb.

Cleve (1901) made a study of the plankton of the Indian Ocean and recorded two species of *Trichodesmium*, viz., *T. Thiebautii* Gomont and *T. erythraeum* Ehrenb.

Iyengar in 1927 recorded *Trichodesmium erythraeum* from the Krusadai Island near Pamban.

In 1935 Boergesen recorded five species from the Coast of Bombay, viz., *Microcoleus chthonoplastes* Thuret, *Sirocoleus Kurzii* Gomont, *Lyngbya majuscula* Harv., *L. confervoides* Ag. and *L. infixa* Frémy. Again in 1936, he recorded eleven species from the coast of Ceylon. (1) *Merismopedia glauca* (Ehrenb.) Näg., (2) *Xenococcus acervatus* Setchell and Gardner, (3) *Dermocarpa olivacea* (Reinsch) Tilden, (4) *D. Leibleiniae* (Reinsch) Bornet and Thuret, (5) *Symploca hydroides* Kütz., (6) *Lyngbya gracilis* Rabenh., (7) *L. sordida* Gomont, (8) *L. majuscula* Harv., (9) *L. infixa* Frémy., (10) *Oscillatoria miniata* (Zanard.) Hauck and (11) *O. corallinae* (Kütz.) Gomont.

*From the Department of Botany, University of Madras.

Gopala Iyer and Sankara Menon (1936) have recorded two blue-green algae, viz., *Trichodesmium erythraeum* Ehrenb., and *T. Thiebautii* Gomont from the Madras Coast.

Dixit (1936) working on the Myxophyceae of Bombay Presidency records ten species which were growing in waters 'containing high percentage of salt, (i.e., sea water)'. They are (1) *Aphanocapsa littoralis* Hansg., var. *macrococca* Hansg., (2) *Anabaena orientalis* Dixit, (3) *Spirulina subsalsa* Oerst., (4) *Sp. major* Kütz., (5) *Oscillatoria sancta* (Kütz.) Gomont, (6) *Oscillatoria acuminata* Gomont, (7) *Lyngbya aestuarii* Leibm., (8) *L. majuscula* Harv., (9) *L. confervoides* Ag. and (10) *Microcoleus chthonoplastes* Thuret.

The above records are mainly from the coast of Bombay or Ceylon. Excepting for the one species recorded by Iyengar and the two by Gopala Iyer and Sankara Menon, practically no work appears to have been done on the Marine Myxophyceae of South India. A large collection of marine blue-green algae was made by Iyengar over a period of several years. These were collected from Pamban, Krusadai Island, Shingle Island, Rameswaram, Dhanushkodi, Hare Island near Tuticorin, Madras and Seven Pagodas. A few forms collected from Karwar in the Bombay Presidency and Galle and Keerimalai in Ceylon are also included in this account. Altogether 32 forms have been recorded in this paper representing 18 genera, 28 species, 1 variety and 3 forms of which 1 species and 2 forms are new.

MYXOPHYCEAE

CHROOCOCCALES Wettstein.

Family CHROOCOCCACEAE Nägeli.

Genus *Aphanocapsa* Nägeli.

- (1) *Aphanocapsa littoralis* Hansgirg.

(Fig. 1)

Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 153, fig. 66b, 1932; Boergesen and Frémy, *Marine Algae of the Canary Islands*, *Kgl. Danske Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 8, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Danske Botanisk Arkiv.*, Bd. IX, n. 7, p. 5, 1939.

Cells densely arranged in an amorphous mucilage, devoid of an individual sheath or tegument, spherical, 3.9—5.2 μ broad; contents blue-green and homogeneous.

On rocks along with other algae, Shingle Island.

Family ENTOPHYSALIDACEAE Geitler.

Genus *Entophysalis* Kützing.

- (2) *Entophysalis granulosa* Kützing.

(Fig. 2)

Kützing, *Phyc. gen.*, p. 177, pl. XVIII, fig. 5, 1843; *Tab. phycologicae*, I, p. 22, pl. 32, fig. 1, 1849; *Sp. Algarum*, p. 225, 1849; Rabenhorst, *Fl. Eur. Alg.*, III, p. 43, fig. 16, 1865; Bornet and Thuret, *Notes algologiques*, fasc. I, pl. 1, 1876; Kirchner, in Engler-Prantl, *Natürliche Pflanzenfam.*, Teil I, Abt. Ia, p. 54, fig. 49g, 1900; Forti in De Toni, *Sylloge Algarum*, V, p. 58, 1907; Tilden, *Myxophyceae in Minnesota Algae—I*, p. 24, p. 1, fig. 33, 1910; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 298, fig. 146, 1932.

Plant mass encrusting, thick; cells *Gloeocapsa*-like, with a thick hyaline sheath; cells without sheath 2.6—3.9 μ diam.

On dead coral-stones near shore, Pamban.

This alga differs from the type in the diameter of the cells being smaller.

CHAMAESIPHONALES Wettstein.

Family PLEUROCAPSACEAE Geitler.

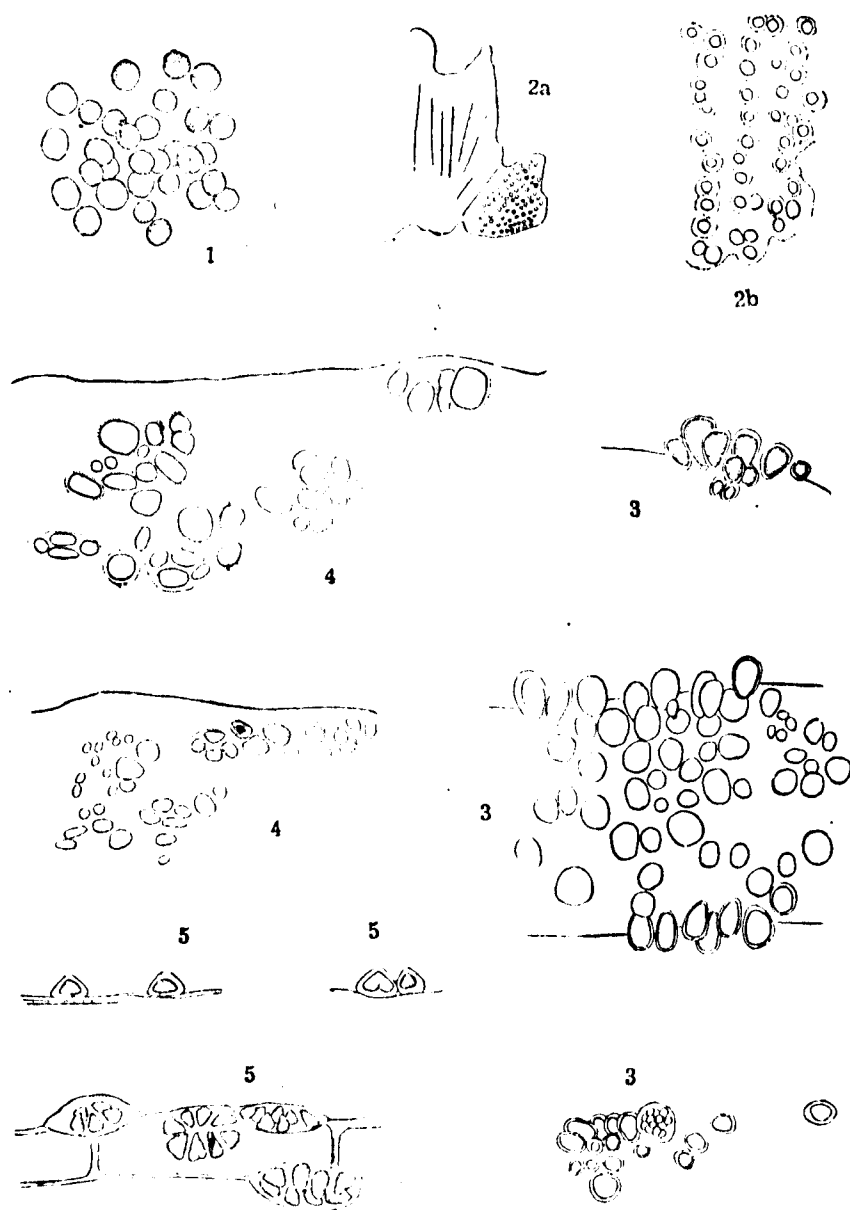
Genus *Xenococcus* Thuret.

- (3) *Xenococcus Cladophorae* (Tilden) Setchell and Gardner.

(Fig. 3)

In Gardner, *New Pacific Coast Marine Algae*, III, *Univ. Calif. Publ., Botany*, Vol. VI, p. 461, pl. 38, fig. 8, 1918; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., *Univ. Calif. Publ., Botany*, VIII, n. 1, p. 33, pl. 4, fig. 8, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 336, fig. 171, 1932; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, 1939.

= *Chlorogloea tuberculosa* Setchell and Gardner, Tilden, *Myxophyceae in Minnesota Algae*—I, p. 46, 1910 (in part).



Figs. 1-5. Fig. 1. *Aphanocapsa littoralis* Hansgirg ($\times 720$). Fig. 2. *Entophysalis granulosa* Kützinger. (a) thallus ($\times 215$). (b) portion of the thallus ($\times 830$). Fig. 3. *Xenococcus Cladophorae* (Tilden) Setchell and Gardner ($\times 385$). Fig. 4. *Xenococcus Chaetomorphae* Setchell and Gardner ($\times 385$). Fig. 5. *Dermocarpa hemisphaerica* Setchell and Gardner ($\times 385$).

Cells forming a more or less single continuous layer, spherical, or pyriform, $9.7-12.9 \mu$ diam., rarely 16.1μ diam; contents pale blue-green, usually homogeneous; endospores present, up to 2.6μ diam.

Growing on *Cladophora* sp., in a salt water pond inside Krusadai Island.

(4) *Xenococcus Chaetomorphae* Setchell and Gardner.

(Fig. 4)

In Gardner, New Pacific Coast Marine Algae, II, *Univ. Calif. Publ., Botany*, VI, p. 436, pl. 36, figs. 2-4, 1918; Setchell and Gardner, The Marine Algae of the Pacific Coast of North America, I—Myxo., *Univ. Calif. Publ., Botany*, VIII, n. 1, p. 35, pl. 2, figs. 2-4, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 336, fig. 173, 1932.

Cells varying in shape and size, spherical, sometimes elongate, up to 19.7μ diam., and about 15μ high; endospores present, $1.3-2.6 \mu$ diam.

On *Chaetomorpha litorea* floating in masses in the estuarine portion of the River Cooum, Madras.

This alga differs from the type in having smaller cells.

Family *DERMOCARPACEAE* Geitler.

Genus *Dermocarpa* Crouan.

(5) *Dermocarpa hemisphaerica* Setchell and Gardner.

(Fig. 5)

In Gardner, New Pacific Coast Marine Algae, II, *Univ. Calif. Publ., Botany*, Vol. VI, p. 438, pl. 37, fig. 21, 1918; Setchell and Gardner, The Marine Algae of the Pacific Coast of North America, I—Myxo., *Univ. Calif. Publ., Botany*, VIII, n. 1, p. 22, pl. 3, fig. 21, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 391, fig. 215, 1932; Frémy, The Marine Algae of the Danish West Indies, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 10, 1939.

Cells epiphytic, solitary, hemispherical, attached by a flat base, $9.7-12.9 \mu$ diam., contents homogeneous; cell-wall thick; reproduction by successive divisions, mostly radial, forming up to 20 endospores.

On *Cladophora* sp., on dead coral rocks, Pamban.

This alga differs from the type in the cells being smaller. It bears a strong resemblance to the genus *Dermocarpella* Lemm., but, as pointed out by Geitler (1932, p. 391), it differs from it in the division being only radial and not transverse. The single cell can easily be mistaken for a *Dermocarpella*.

HORMOGONALES Wettstein.

Family NOSTOCHOPSIDACEAE Geitler.

Genus *Mastigocoleus* Lagerheim.

(6) *Mastigocloeus testarum* Lagerheim.

(Fig. 6)

Bornet and Flahault, Rev. Nost. Hétéro., III, Ann. sci. nat., sér. 7, Bot., V, p. 54, 1887; Kirchner, *Schizophyceae* in Engler-Prantl, *Natürliche Pflanzenfam.*, Teil I, Abt. 1a, p. 81, fig. 58a. 1900; Forti, in De Toni, *Sylloge Algarum*, V, p. 564, 1907; Tilden, *Myxophyceae* in *Minnesota Algae—I*, p. 237, pl. XIV, fig. 12. 1910; Borzi, *Studi Sulle Mixoficee*, p. 140, 1916; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., Univ. Calif. Pub., Botany, Vol. VIII, n. 1, p. 111, 1919; Frémy, *Les Stigonemacées de la France*, *Revue Algologique*, V, p. 154, fig. 5, 1930; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 473, fig. 284, 1932.

Filaments variously curved or bent, branched, branches of two kinds, the one cylindrical and the other tapering and ending in a hair 3.9–6 μ broad; sheath thin, firm and hyaline; trichome single within a sheath, usually 3.9–5.2 μ broad, rarely thinner and 2.6 μ broad, unstricted at the cross-walls; cells usually longer than broad, up to 10.4 μ long, with homogeneous, light blue-green contents; heterocysts terminal or lateral, rarely intercalary, 6.6–7.9 μ broad and 6.6–10.5 μ long; hormogones present.

On shells, Krusadai Island.

The formation of the hormogones was observed in the Krusadai material. A few cells below the hair portion become meristematic and begin to divide. These cells become slightly richer in contents and definitely broader than the remaining cells of the trichome above and below it. Soon the terminal hair portion is thrown off, and the meristematic cells which now occupy the distal end of the trichome continue to divide and finally form a row of about 10–20

short barrel-shaped cells with rich contents. This terminal portion becomes separated from the lower portion through the formation of a disc of intercellular substance or by the death of one or more cells and forms a hormogone which ultimately escapes out of the sheath at the apex of the filament. These hormogones are 2.6–3.9 μ broad. Setchell and Gardner (1919, p. 110) also have

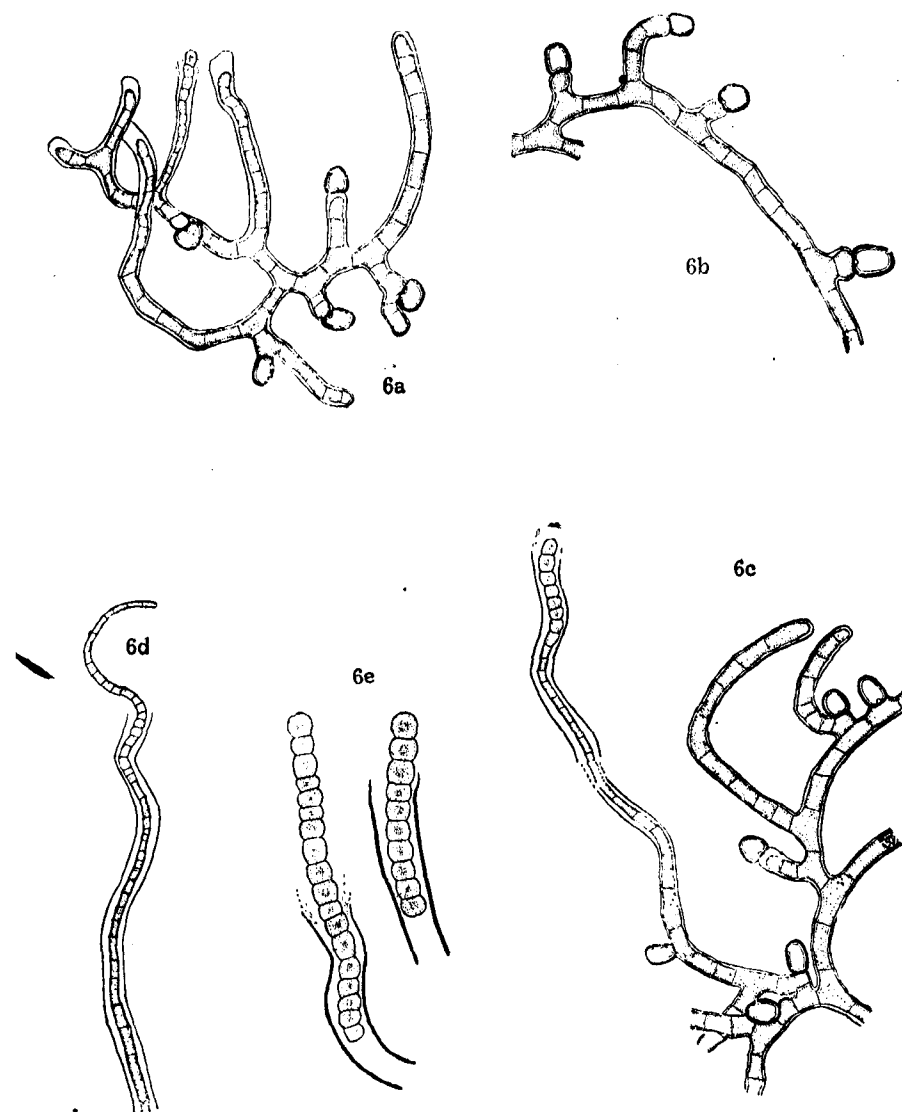


Fig. 6. *Mastigocloeus testarum* Lagerheim. (a) A typical filament ($\times 475$). (b) Portion of a well grown filament ($\times 475$). (c) Portion of a well grown filament ($\times 475$). (d) End portion of a filament showing the hair ($\times 475$). (e) Hormogones ($\times 725$).

recorded that hormogones are formed in their Pacific coast material from the branches which end in hairs. But no details are given regarding their mode of formation.

Family MASTIGOCLADACEAE Geitler.

Genus *Brachytrichia* Zanardini.

(7) *Brachytrichia Balani* (Lloyd) Bornet and Flahault

(Fig. 7)

Rev. Nost. Hétéro., II, Ann. sci. nat., sér 7, Bot., IV, p. 372, 1886; Kirchner, in Engler-Prantl, *Natürliche Pflanzenfam.* Teil I, Abt. 1a, p. 90, fig. 60c, 1900; Forti, in De Toni, *Sylloge Algarum*, V, p. 679, 1907; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 554, figs. 347, 348, 1932; Boergesen and Frémy, *Marine Algae of the Canary Islands*, Kgl. Danske Videns. Selsk., Biol. Medd., XII, n. 5, p. 37, 1936.

= *Hormactis Balani*, Thuret, Essai de la classification des Nostichinées, Ann. sci. nat., sér. VI, Bot., I, p. 382, 1875; Bornet and Thuret, *Notes algologiques*, fasc. II, p. 172, pl. 43 and 44, 1880.

= *Brachytrichia Quoyi* (Ag.) Bornet and Flahault, Rev. Nost. Hétéro., II, Ann. sci. nat., sér. 7, Bot., IV, p. 373, 1886; Forti, in De Toni, *Sylloge Algarum*, V, p. 680, 1907; Tilden, *Myxophyceae* in *Minnesota Algae—I*, p. 294, pl. 20, fig. 18, 1910; Frémy, *The Marine Algae of the Danish West Indies*, Dansk Botanisk Arkiv., Bd. IX, no. 7, p. 40, 1939.

= *Brachytrichia maculans* Gomont, Forti in De Toni, *Sylloge Algarum*, V, p. 679, 1907.

= *Brachytrichia affinis* Setchell and Gardner, in Gardner, *New Pacific Coast Marine Algae*, III, Univ. Calif. Publ., Botany, VI, p. 475, pl. 41, figs. 27 and 28, 1918; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., Univ. Calif. Publ., Botany, VIII, n. 1, p. 112, pl. 7, figs. 27 and 28, 1919.

= *Brachytrichia Codii* Gardner, *Tahitian Algae*, Univ. Calif. Publ., Botany, XII, n. 5, p. 66, 1926.

Plant mass up to 1-1½ cm., diam., attached, *Nostoc*-like, mostly hollow inside, blue-green in colour; filaments arranged close to each other densely; erect filaments arranged more or less

parallel, the apex often terminating in a hair; trichome 3.9-5.2 μ broad, when old 2.6 μ broad, apical hair up to 1.3 μ broad; cells

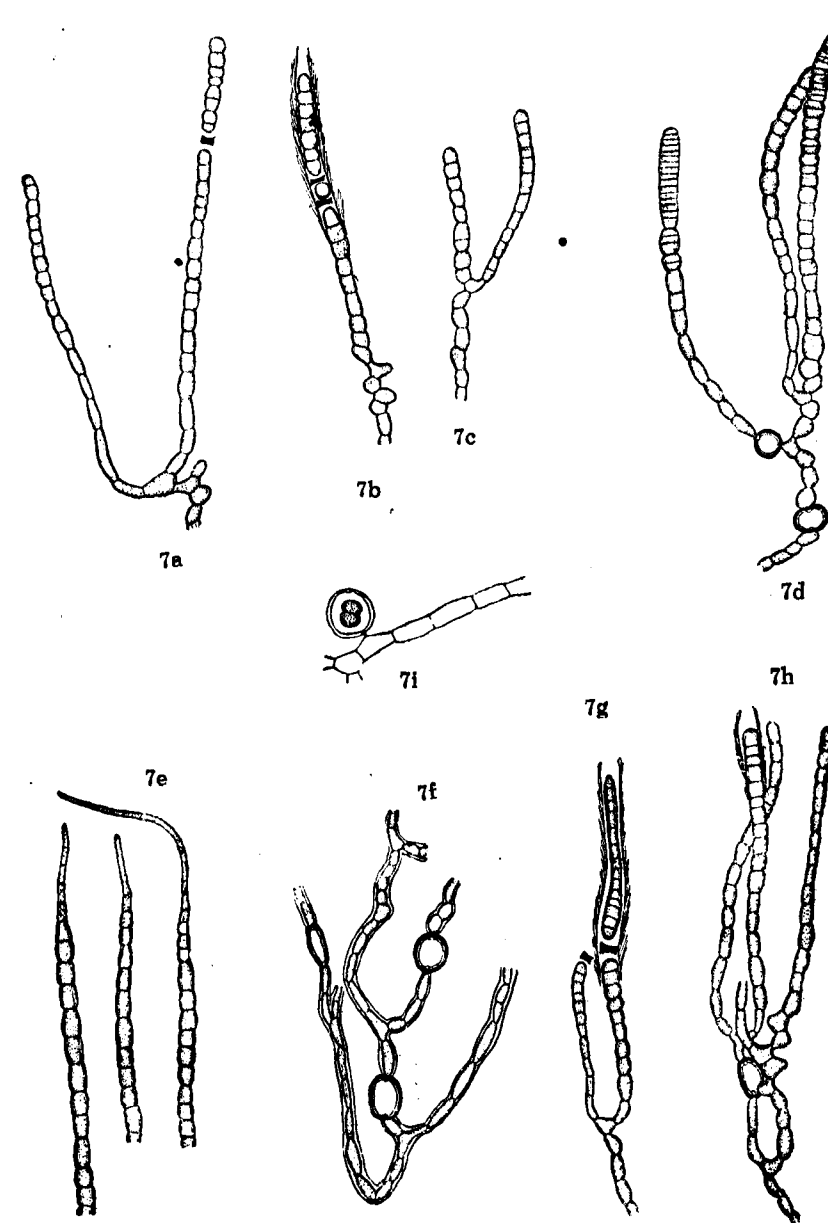


Fig. 7. *Brachytrichia Balani* (Lloyd) Born. et Flah. (a), (c), (d) and (h) Filaments showing reverse V-shaped branching ($\times 575$). (b) and (g) Filaments showing hormogones ($\times 575$). (e) Ends of filaments showing the hairs ($\times 575$). (f) Older portions of the filament ($\times 575$). (i) A two-celled germling formed by a heterocyst ($\times 575$).

barrel-shaped, up to twice as long as broad, cells near the apex often shorter than broad, cells of the hair up to 6 times as long as broad; sheath often indistinct and hyaline, sometimes thick and somewhat lamellated near the apex before hormogone formation; heterocysts intercalary; spherical to subspherical, $6.6-10.5 \mu$ broad, as long as broad or rarely up to twice as long as broad; hormogones present.

Growing gregariously on stones below bridge at Pamban.

This alga was found growing gregariously in the intertidal zone and formed a carpet as it were on the upper side of large stones below the railway bridge at Pamban. These remained exposed for a long time during low tide.

Formation of hormogones has been observed in this alga. Hormogones are usually abstricted from the apices of piliferous branches and are 6-20 cells long (Fig. 7 b and g). A single case of germination of the heterocyst has been recorded in this Pamban material (Fig. 7 i).

Family RIVULARIACEAE Rabenhorst.

Genus *Dichothrix* Zanardini

(8) *Dichothrix Bauermana* (Grun.) Bornet and Flahault.

(Fig. 8)

Rev. Nost. Hétéro., I, *Ann. sci. nat.*, sér. 7, Bot., III, p. 375, 1886; Forti, in De Toni, *Sylloge Algarum*, V, p. 640, 1907; Tilden, *Myxophyceae in Minnesota Algae*—I, p. 276, pl. XVIII, fig. 18, 1910; Getiler, *Cyanophyceae in Rabenhorst's Kryptogamenflora*, XIV, p. 586, fig. 367, 1932.

= *Schizosiphon sociatus* Suringar, Rabenhorst, *Fl. Eur. Alg.*, II, p. 239, 1865.

= *Mastigonema Bauermanum* Grunow, in Rabenhorst, *Fl. Eur. Alg.*, II, p. 227, 1865.

= *Schizosiphon Bauermanum* Grunow, in Rabenhorst, *Fl. Eur. Alg.*, II, p. 238, 1865; Wolle, *Freshwater Algae*, Bull. Torrey Club, VI, p. 284, 1879.

= *Calothrix Bauermana* (Grunow) Hansgirg, *Prodromus der Algenflora von Böhmen*, I, pt. 2, p. 53, 1893.

Filaments $9.2-17 \mu$ broad; sheath thin or sometimes thick and gelatinous, not lamellated, hyaline or slightly yellowish; trichome swollen at the base, $6.6-9.2 \mu$ broad and above

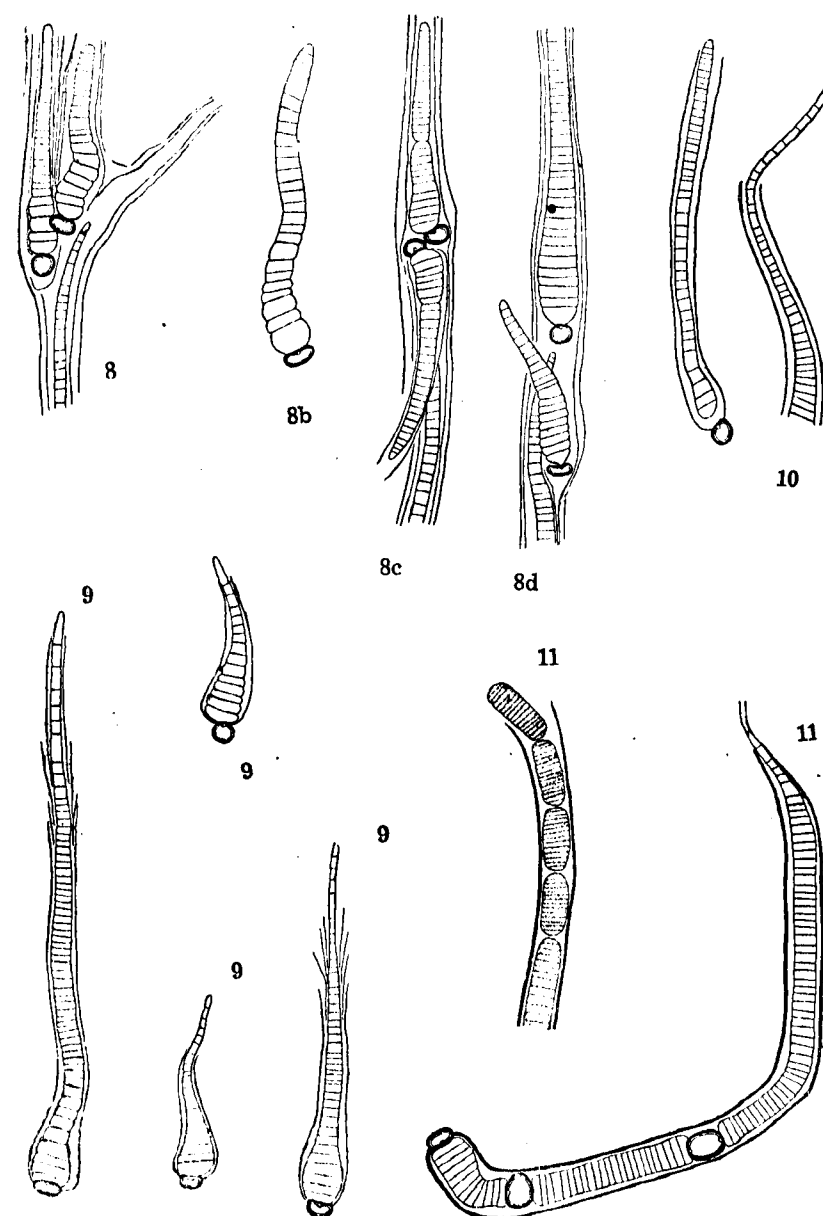


Fig. 8-11. Fig. 8. *Dichothrix Bauermana* (Grun.) Born. et Flah. ($\times 550$). Fig. 9. *Calothrix Contarenii* Born. et Flah. ($\times 375$). Fig. 10. *Calothrix scopulorum* (W. et M.) Ag. ($\times 375$). Fig. 11. *Calothrix crustaceae* Thuret ($\times 225$).

3.9-6.6 μ broad, sometimes constricted at the cross-walls, tapering towards the apex; cells up to $\frac{1}{2}$ as long as broad; heterocysts basal, not intercalary, spherical to subspherical.

Growing on the sides of the Madras Harbour walls.

This alga resembles the type in all respects except that the filaments are thinner and the trichome is slightly swollen at the base (cf. Geitler, 1932, fig. 367).

Genus *Calothrix* Ag.

(9) *Calothrix Contarenii* Bornet and Flahault.

(Fig. 9)

Rev. Nost. Hétéro. I, *Ann. sci. nat.*, sér. 7, Bot., III, p. 355, 1886; Forti, in De Toni, *Sylloge Algarum*, V, p. 610, 1907; Tilden, *Myxophyceae in Minnesota Algae I*, p. 259, pl. XVI, fig. 13, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America, I—Myxo.*, *Univ. Calif. Publ., Botany*, VIII, n. 1, p. 97, 1919; Geitler, *Cyanophyceae*, in Rabenhorst's *Kryptogamenflora*, XIV, p. 600, 1932.

= *Rivularia Contarenii* Zanardini, Meneghini, *Monographia Nostochinearum Italicarum addito specimen de Rivulario*, *R. Acad. Sci.*, p. 141, pl. 17, fig. 2, 1841.

= *Rivularia Chaetopus* Kützing, *Phyc. gen.*, p. 234, 1843; *Sp. Algarum*, p. 328, 1849; *Tab. phycologicae*, II, p. 14, pl. 48, fig. 5, 1850.

= *Mastichonema Contarenii* Kützing, Kützing, *Sp. Algarum*, p. 326, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 227, 1865.

Plant mass crustaceous, bluish black; filaments 11.8 — 19.6 μ broad; sheath thin, hyaline, rarely brownish; trichome straight or bent, broadened at the base, (6.6 —) 9.2 — 13.1 (— 15.7) μ broad, with an attenuated hair; cells up to half as long as broad; heterocysts only basal.

On dead coral stones, Pamban and Krusadai Island.

This alga differs from the type in having slightly broader filaments and trichomes.

(10) *Calothrix scopulorum* (W. and M.) Ag.

(Fig. 10)

Harvey, *Phyc. Brit.*, I, pl. 58, 1846; *Nereis Boreali-Americana*, III, p. 105, 1858; Steinheil, *Cryptogames de Barbarie*, *Ann. sci. nat.*, sér. 2, Bot., I, p. 289, 1834; Thuret, *Essai de la classification des Nostochinées*, *Ann. sci. nat.*, sér. 6, Bot., I, p. 381, 1875; Bornet and Thuret, *Notes algologiques*, fasc. II, p. 159, 1880; Bornet and Flahault, *Rev. Nost. Hétéro.*, I, *Ann. sci. nat.*, sér. 7, Bot., III, p. 353, 1886; Forti, in De Toni, *Sylloge Algarum*, V, p. 608, 1907; Tilden, *Myxophyceae in Minnesota Algae-I*, p. 258, pl. 25, figs. 11 and 12, 1910; Setchell and Gardner, *The Marine Algae of Pacific Coast of North America, I, Myxo.*, *Univ. Calif. Publ., Botany*, VIII, n. 1, p. 96, 1919; Geitler, *Cyanophyceae in Rabenhorst's Kryptogamenflora*, XIV, p. 600, fig. 374, 1932; Boergesen and Frémy, *Marine Algae of the Canary Islands*, IV, *Cyanophyceae*, *Kgl. Danske Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 33, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 35, 1939.

= *Schizosiphon scopulorum* Kützing, *Phyc. gen.*, p. 233, 1843; *Sp. Algarum*, p. 329, 1849; *Tab. phycologicae*, II, p. 15, pl. 50, fig. 5, 1850; Rabenhorst, *Fl. Eur. Alg.*, II, p. 240, 1865.

= *Schizosiphon rupicola* Kützing, *Phyc. gen.*, p. 223, 1843; *Sp. Algarum*, p. 328, 1849; *Tab. phycologicae*, II, p. 14, pl. 48, fig. 5, 1850.

= *Schizosiphon gregarius* Kützing, *Sp. Algarum*, p. 329, 1849; *Tab. phycologicae*, II, p. 15, pl. 50, fig. 4, 1850.

Filaments erect, slightly thickened at the base, forming one expanded mass 7.9 — 13.1 μ broad; sheath hyaline, rarely brownish, lamellated; trichomes (5.3 —) 6.5 — 9.1 μ broad, prolonged into a long hair; cells shorter than broad; heterocysts basal only, 6.55 — 10.4 μ broad, subspherical, rarely oblong.

On dead coral stones, Krusadai Island.

The filaments and trichomes in this alga are slightly thinner than in the type.

(11) *Calothrix crustacea* Thuret.

(Fig. 11)

In Bornet and Thuret, *Notes algologiques*, fasc. I, p. 13-16, pl. 4, 1876; Bornet and Flahault, *Rev. Nost. Hétéro.*, I, *Ann. sci.*

nat., sér. 7, Bot., III, p. 359, 1886; Forti, in De Toni, *Sylloge Algarum*, V, p. 613, 1907; Tilden, *Myxophyceae*, in *Minnesota Algae*, I, p. 264, pl. 17, figs. 2-6, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I-Myxo., Univ. Calif. Publ., Botany, VIII, n. 1, p. 99, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 601, fig. 375b, 1932; Boergesen and Frémy, *Marine Algae of the Canary Islands*, Kgl. Danske Vidensk. Selsk., Biol. Medd., XII, n. 5, p. 34, 1936; Frémy, *The Marine Algae of the Danish West Indies*, Dansk Botanisk Arkiv., Bd. IX, n. 7, p. 35, 1939.

= *Schizosiphon dilatatus* Kützinger, *Sp. Algarum*, p. 330, 1849.

= *Schizosiphon lutescens* Kützinger, *Phyc. gen.*, p. 233, 1843; *Sp. Algarum*, p. 327, 1849; *Tab. phycologicae*, II, p. 14, pl. 48, fig. 1, 1850; Rabenhorst, *Fl. Eur. Alg.*, II, p. 240, 1865.

= *Schizosiphon lasiopus* Kützinger, *Phyc. gen.*, p. 234, 1843; *Sp. Algarum*, p. 328, 1849; *Tab. phycologicae*, II, p. 15, pl. 49, fig. 5, 1850; Rabenhorst, *Fl. Eur. Alg.*, II, p. 241, 1865.

Filaments brownish, erect or slightly bent, sometimes slightly thickened at the base, $15.7 - 19.7 \mu$ broad; sheath somewhat thick, homogeneous, sometimes coloured brownish; trichome $10.5 - 13.1 \mu$ broad, prolonged into a delicate hair, mostly torulose; cells shorter than broad; heterocysts basal and intercalary, $10.5 - 14.4 \mu$ broad; hormogones present.

Growing on shells among other algae, Krusadai Island.

Family MICROCHAETACEAE Lemmermann.

Genus *Microchaete* Thuret.

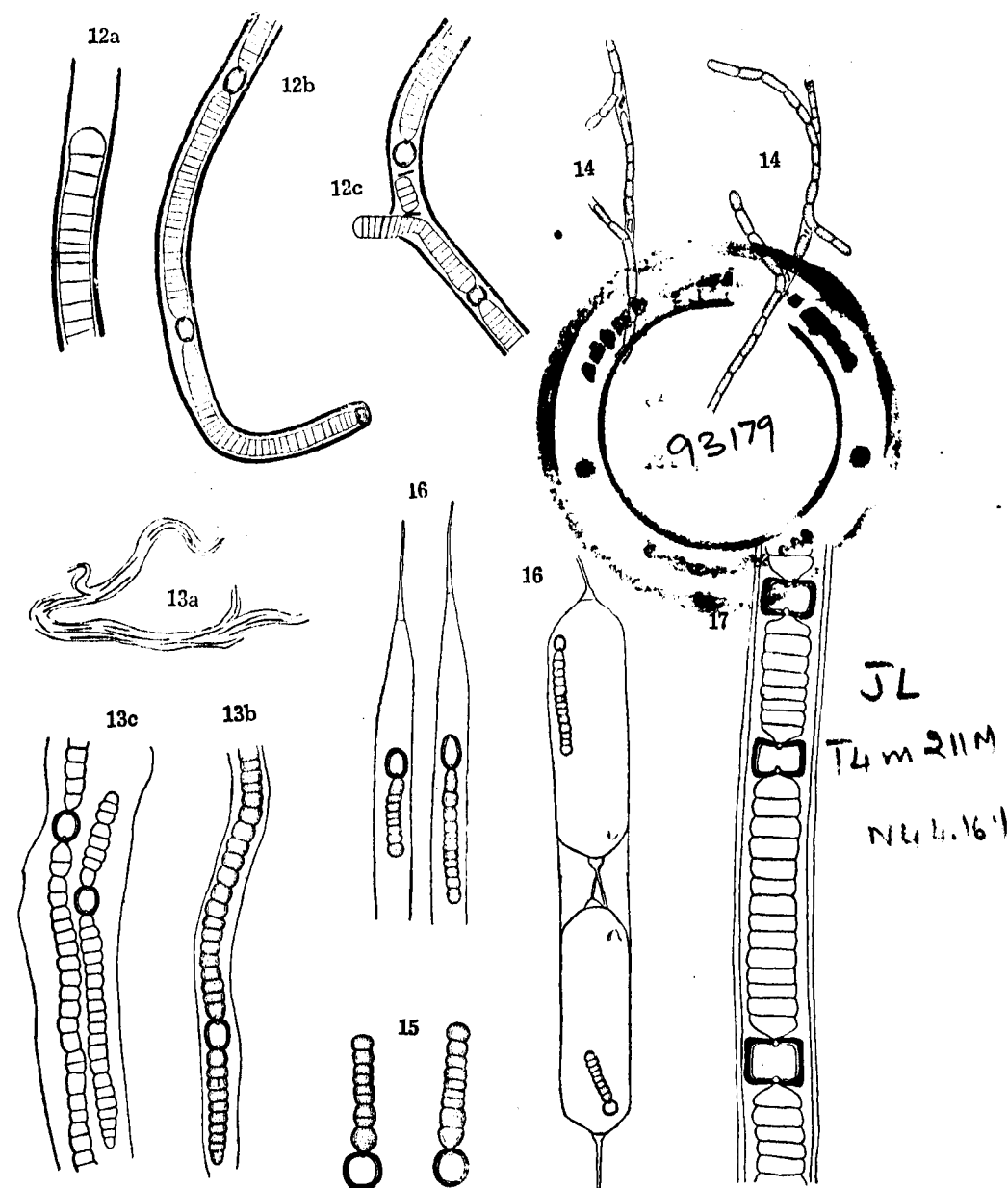
(12) *Microchaete aeruginea* Batters.

(Fig. 12)

Journal of Botany, XXX, p. 86, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 486, 1907; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 666, 1932.

Filaments variously bent, $12.9 - 16.1 \mu$ broad; branching rare, branches single; sheath unconstricted at the cross-walls, apex slightly broader, blue-green in colour; cells usually shorter than broad, sometimes as long as broad; heterocysts basal and intercalary, as broad as the vegetative cell.

On dead coral stones near high tide along with *Cladophora*, Shingle Island.



Figs. 12-17. Fig. 12. *Microchaete aeruginea* Batters. (a) Portion of a filament with a basal heterocyst and intercalary heterocysts ($\times 215$). (b) Apical portion of a filament ($\times 395$). (c) A portion of the filament showing a false branch ($\times 215$). Fig. 13. *Hormothamnion Enteromorphoides* Grun. (a) Habit ($\times 50$). (b) Single trichome enclosed in a sheath ($\times 490$). (c) Two

The alga differs from the type in the broader filaments and trichomes. Branching is seen only very rarely and is single.

Genus *Hormothamnion* Grunow.

(13) *Hormothamnion Enteromorphoides* Grunow.

(Fig. 13)

Bornet and Flahault, Rev. Nost. Hétéro., IV, Ann. sci. nat., sér. 7, Bot., VII, p. 260, 1888; Kirchner, Schizophyceae in Engler-Prantl, Natürliche Pflanzenfam., Teil 1, Abt. 1a, p. 70, fig. 56j, 1900; Forti, in De Toni, Sylloge Algarum, V, p. 486, 1907; Tilden, Myxophyceae in Minnesota Algae-I, p. 205, pl. 10, fig. 13, 1910; Geitler, Cyanophyceae, in Rabenhorst's Kryptogamenflora, XIV, p. 677, fig. 436, 1932; Boergesen and Frémy, Marine Algae of the Canary Islands, Kgl. Danske Vidensk. Selsk., Biol. Medd., XII, n. 5, p. 40, 1936; Frémy, The Marine Algae of the Danish West Indies, Dansk Botanisk Arkiv., Bd. IX, n. 7, p. 44, 1939.

Plant mass caespitose, branched, branches tapering; sheath thin and hyaline, sometimes gelatinizing; trichome one to many in a sheath, 5.3 — 6.5 μ broad, torulose; cells barrel-shaped, shorter or longer than broad, contents light blue-green and homogeneous; heterocysts usually as broad as the vegetative cell or sometimes broader, 6.5 — 7.8 (—9.2) μ broad, single and intercalary.

On *Spyridia filamentosa*, Krusadai Island.

Family SCYTONEMATACEAE Rabenhorst.

Genus *Plectonema* Thuret.

(14) *Plectonema terebrans* Bornet and Flahault.

(Fig. 14)

Gomont, Mono. Oscillariées, Ann. sci. nat., sér. 7, Bot., XVI, p. 102, 1892; Forti, in De Toni, Sylloge Algarum, V, p. 497, 1907; Tilden, Myxophyceae, in Minnesota Algae-I, p. 209, pl. XI, fig. 6,

trichomes enclosed in a sheath ($\times 490$). Fig. 14. *Plectonema terebrans* Born. et Flah. ($\times 490$). Fig. 15. *Richelia intracellularis* Schmidt ($\times 490$). Fig. 16. *Richelia intracellularis* Schmidt forma ($\times 395$). Fig. 17. *Nodularia spumigena* Mertens var. *major* (Kütz.) Bornet and Flahault. ($\times 490$).

1910; Geitler, Cyanophyceae in Rabenhorst's Kryptogamenflora, XIV, p. 683, fig. 437a, 1932.

Filaments slender, elongate, flexuous; false branching present, single; sheath very thin, hyaline; trichome about 1.3 μ broad, unconstricted or slightly constricted at the cross-walls cells 2-3 times as long as broad; apex rounded; cell-contents pale blue-green.

On the shells of living Gastropods, Krusadai Island.

This alga differs from the type in the absence of a granule on either side of the cross-wall.

Family NOSTOCACEAE Kützing.

Genus *Richelia* Schmidt.

(15) *Richelia intracellularis* Schmidt.

(Fig. 15)

Lemmermann, Die Algenflora der Sandwich-inseln, p. 622, pl. 7, fig. 1, 1896; Forti, in De Toni, Sylloge Algarum, V, p. 480, 1907; Karsten, Das Indische Phytopl. Wiss. Ergeb. deutsch. Tiefsee Exp., Bd. 2, Teil II, Leif, 1907; Tilden, Myxophyceae in Minnesota Algae-I, p. 201, pl. 10, fig. 8, 1910; Geitler, Cyanophyceae, in Rabenhorst's Kryptogamenflora, XIV, p. 804, fig. 513, 1932.

Trichome up to 51.5 μ long, 4.9 — 6.5 μ broad, straight, constricted at the cross-walls, cross-walls not granulated; cells half as long as broad; heterocysts terminal, at one end, 7.9 — 10.5 μ (—11.6) μ broad, spherical, single.

In the cells of the diatom *Rhizosolenia*, in the plankton of the Madras Coast, (leg., R. Subrahmanian).

(16) *Richelia intracellularis* Schmidt forma.

(Fig. 16)

Length of trichome upto 40 μ ; breadth of the cells 2.6 — 3.9 μ ; heterocysts about 5.2 μ broad.

In the cells of the diatom *Rhizosolenia* in the Plankton of the Madras coast (leg., R. Subrahmanian).

This form differs from the type in the trichomes being thinner and shorter and the heterocyst smaller.

Genus *Nodularia* Mertens.

(17) *Nodularia spumigena* Mertens var. *major* (Kütz.) Bornet and Flahault.

(Fig. 17)

Rev. Nost. Hétéro. IV, Ann. sci. nat., sér. 7, Bot., VII, p. 247, 1888; Forti in De Toni, *Sylloge Algarum*, V, p. 435, 1907; Tilden, *Myxophyceae* in *Minnesota Algae-I*, p. 185, 1910; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 867, 1932.

= *Spermosira major* Kützing, Kützing, *Sp. Algarum*, p. 295, 1849; *Tab. phycologicae*, I, p. 54, pl. 100, fig. 5, 1849;

= *Spermosira litorea* Crassior, Rabenhorst, *Fl. Eur. Alg.*, II, p. 186, 1865.

Filaments intertwining, forming a large, sometimes gelatinous, mass, 14.4-18.3 μ broad; sheath thin, sometimes lamellated, hyaline; trichome torulose (7.9) 9.2-13.1 μ broad; cells discoid, short, 3.9-6.5 rarely 9.2 μ long; heterocysts usually as broad as the trichome, 10.5-14.4 μ broad, shorter than long or sometimes as long as broad, 7.9-10.5 (-13.1) μ long; spores not observed.

Washed ashore, Krusadai Island, Shingle Island, Kuntukal point near Famban and Hare Island.

The alga could not be identified definitely as the spores were not observed in any of the collections. It is referred tentatively to *N. spumigena* var. *major* with which it agrees generally in other respects.

Family *OSCILLATORIACEAE* (Gray) Kirchner.

Genus *Spirulina* Turpin.

(18) *Spirulina labyrinthiformis* (Menegh.) Gomont

(Fig. 18)

Mono. Oscillarées, Ann. sci. nat., sér. 7, Bot., VI, p. 255, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 215, 1907; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 928, 1932; Frémy, Les Lyngbyées de la Normandie, *Mem. Soc. Arch.*

Hist. Natur. dept. Manche, XLVII, 77, 1935; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vednsk. Selsk., Biol. Medd.*, XII, n. 5, p. 31, 1936.

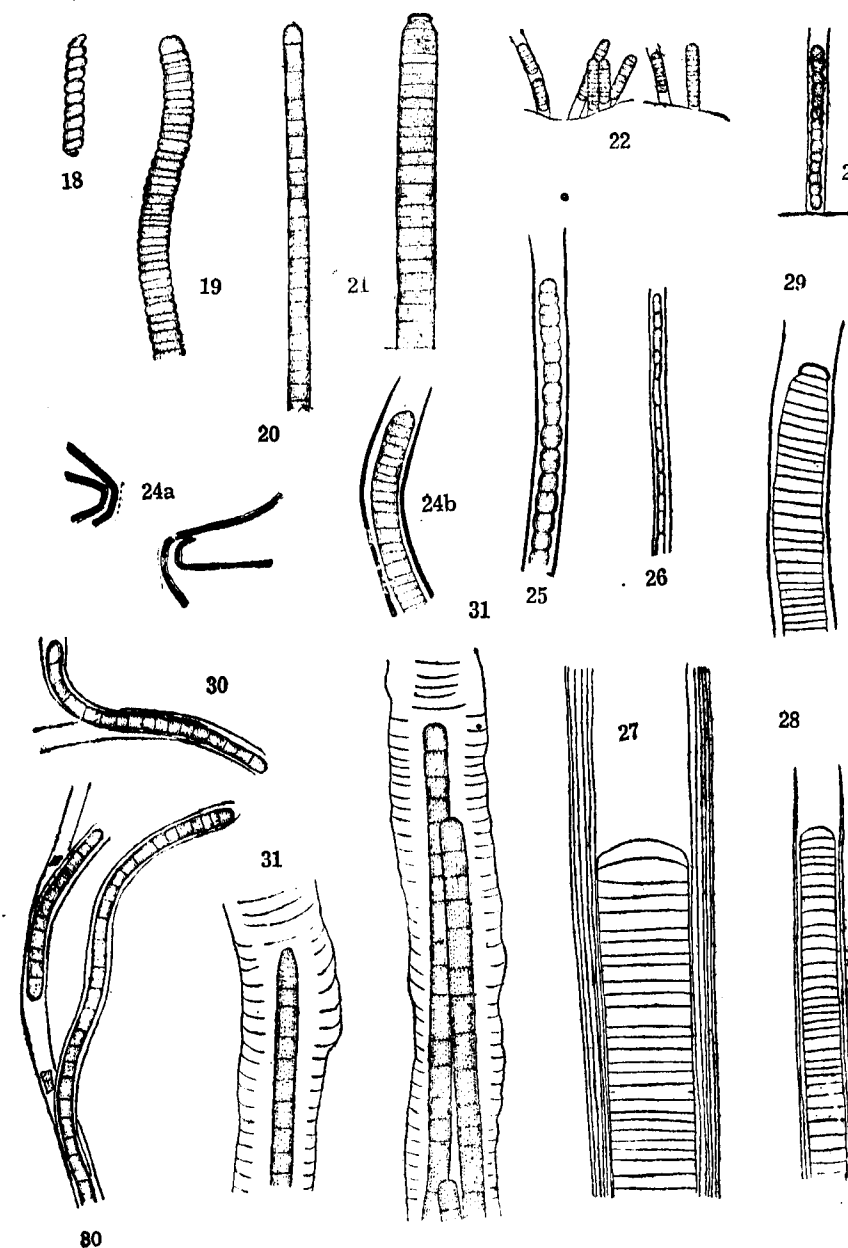


Fig. 18-31. Fig. 18. *Spirulina labyrinthiformis* (Menegh.) Gomont ($\times 790$). Fig. 19. *Oscillatoria nigro-viridis* Thwaites ($\times 340$). Fig. 20. *Oscillatoria subuliformis* Kützing ($\times 340$). Fig. 21. *Oscillatoria erythraea* (Ehrb.) Geitler ($\times 450$). Fig. 22. *Lyngbya Chaetomorphae* sp. nov. ($\times 790$).

Trichome about 1 μ broad, light blue-green, regularly coiled, spirals very close to each other, 2.6-3 μ broad.

In backwaters at Ennore, near Madras (spirals 2.6 μ br.) and on shells amidst other algae, Krusadai Island (spirals 2.6-3 μ br.).

(19) *Spirulina subtilissima* Kützing.

Phyc. gen., p. 183, 1843; *Sp. Algarum*, p. 235, 1849; *Tab Phycologicae*, I, p. 26, pl. 37, fig. 6, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 93, 1865; Hansgirg, *Prodromus der Algenflora von Böhmen*, II, p. 120, 1892; Gomont, *Mono. Oscillarées*, *Ann. sci. nat., sér. 7, Bot.*, XVI, p. 252, pl. 7, fig. 30, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 212, 1907; Tilden, *Myxophyceae in Minnesota Algae—I*, p. 88, 1910; Geitler, *Cyanophyceae in Rabenhorst's Kryptogamenflora*, XIV, p. 929, 1932; Frémy, *Les Lyngbyées de la Normandie*, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, XLVII, p. 75, 1935; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 30, 1936.

Trichome light blue-green, loosely coiled; spirals about 1.3 μ broad.

Forming a layer of closely entangled trichomes on *Neomeris annulata*, in a laboratory culture of some material got from Pamban.

Genus *Oscillatoria* Vaucher.

(20) *Oscillatoria nigro-virides* Thwaites.

(Fig. 19)

In Harvey, *Phyc. Brit.*, pl. 251a, 1846-51; Gomont, *Mono. Oscillarées*, *Ann. sci. nat., sér. 7, Bot.*, XVI, p. 217, pl. 6, fig. 20, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 161, 1907; Tilden,

Fig. 23. *Lyngbya Gardneri* (Setchell and Gardner) Geitler ($\times 790$). Fig. 24. *Lyngbya Baculum* Gomont forma. (a) Habit ($\times 45$). (b) Apical portion of a filament ($\times 450$). Fig. 25. *Lyngbya Holdenii* Forti ($\times 450$). Fig. 26. *Lyngbya Nordgaardii* Wille ($\times 790$). Fig. 27. *Lyngbya majuscula* Harvey ($\times 340$). Fig. 28. *Lyngbya confervoides* Ag. ($\times 450$). Fig. 29. *Lyngbya semiplena* Ag. ($\times 450$). Fig. 30. *Symploca hydroides* Kütz. f. indica f. nov. ($\times 450$). Fig. 31. *Microcoleus chthonoplastes* Thuret ($\times 450$).

Myxophyceae, in *Minnesota Algae—I*, p. 69, pl. 4, fig. 12, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I, Myxo., *Univ. Calif. Publ., Botany*, VIII n. 1, p. 62, 1919; Geitler, *Cyanophyceae in Rabenhorst's Kryptogamenflora*, XIV, p. 942, fig. 597c, 1932; Frémy, *Les Lyngbyées de la Normandie*, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 56, 1935; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vednsk. Selsk., Biol. Medd.*, XII, n. 5, p. 27, fig. 4, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkv.*, Bd. IX, n. 7, p. 32, 1939.

Trichomes blue-green, flexuous, 10.5-11.7 μ broad, constricted at the cross-walls; no granulation at the cross-walls; apex of the trichome round; cells 2.6-3.9 μ long.

Plankton, Madras coast.

This differs from the type in the shorter cells and in the absence of granulation at the cross-walls.

(21) *Oscillatoria subuliformis* Kützing.

(Fig. 20)

Thuret, *Essai de la classification des Nostochinées*, *Ann. sci. nat., sér. 6, Bot.*, I, p. 378, 1875; Gomont, *Mono. Oscillarées*, *Ann. sci. nat., sér. 7, Bot.*, XVI, p. 226, pl. 7, fig. 10, 1892; Geitler, *Cyanophyceae*, in *Rabenhorst's Kryptogamenflora*, XIV, p. 949, fig. 603b, 1932; Frémy, *Les Lyngbyées de la Normandie*, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 61, 1935; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 29, fig. 7, 1936.

=*Oscillaria subuliformis* Thwaites in Harvey, *Phyc. Brit.*, III, pl. 251b, 1851; Rabenhorst, *Fl. Eur. Alg.*, II, p. 114, 1865; Cooke, *Brit. Freshwater algae*, p. 251, pl. 98, fig. 3, 1882-84;

=*Oscillatoria subuliformis* (Thwaites) Gomont, Forti in De Toni, *Sylloge Algarum*, V, p. 176, 1907.

Trichomes greenish, straight, unconstricted at the cross-walls, cross-walls not granulated, 6.5-7.9 μ broad; cells shorter or longer than broad, 3.9-7.8 μ long.

Plankton, Madras Coast.

(22) *Oscillatoria erythraea* Ehrenbg.

(Fig. 21)

Geitler, *Cyanophyceae*, in Rabenhorst's *Kryptogamenflora*, XIV, p. 968, fig. 617a and b, 1932.

= *Trichodesmium erythraeum* Ehrenberg, Rabenhorst; *Fl. Eur. Alg.*, II, p. 161, 1865; Forti in De Toni *Sylloge Algarum*, V, p. 202, 1907; Tilden, *Myxophyceae* in *Minnesota Algae*—I, p. 84, pl. 4, fig. 40, 1910.

= *Oscillaria erythraea* Kützinger, *Phyc. gen.*, p. 188, 1843.

= *Trichodesmium Ehrenbergii* Montagne, *Ann. sci. nat.*, sér. 3, Bot., II, p. 360, pl. 10, 1844; Kützinger, *Sp. Algarum*, p. 286, 1849; *Tab. phycologicae*, I, p. 149, pl. 91, fig. 3, 1848.

= *Trichodesmium Hindsii* Montagne in *Ann. sci. nat.*, sér. 3, Bot., II, p. 360, pl. 10, fig. d, 1844; Kützinger, *Sp. Algarum*, p. 287, 1849; *Tab. phycologicae*, I, p. 49, pl. 91, fig. 4, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 16, 1865.

Trichomes arranged in free-swimming bundles, (7-9-) 9.2-11.8 μ broad, not constricted at the cross-walls; apex slightly tapering, with a thick membrane; cells half to as long as broad, 2.6-5.3 μ long.

This alga differs from the type in the trichomes being not constricted at the cross-walls.

Plankton, Madras coast, Pamban, Krusadai Island.

Genus *Lyngbya* Ag.

Section *Heteroleibleinia* Geitler.

(23) *Lyngbya Chaetomorphae* sp. nov.

(Fig. 22)

Trichomes erect, light blue-green in colour, short, up to 26.2 μ long, usually about 1.3 μ broad; apex rounded, not calyptrate, not constricted at the cross-walls; cross-walls not granulated; sheath very thin and hyaline, closely investing the trichome; cells shorter than broad.

Along with *Xenococcus Chaetomorphae* on *Chaetomorpha litorea* floating in masses in the estuarine region of the River Cooum, Madras.

This alga resembles *Lyngbya Kützingerii* var. *minor*, but differs from it in the trichomes being thinner and much shorter.

(24) *Lyngbya Gardneri* (Setchell and Gardner) Geitler.

(Fig. 23)

Cyanophyceae, in Rabenhorst's *Kryptogamenflora*, XIV, p. 103, 1932.

Filaments epiphytic, straight and slightly flexuous, about 2.6 μ broad; sheath thin and hyaline; trichome light blue-green 1.3-2 μ broad, slightly constricted at the cross-walls, cross-walls not granulated; cells slightly longer than broad; apical cell rounded, not calyptrate.

On *Lyngbya majuscula* Harvey, washed ashore at Dhanuskodi.

Section *Leibleinia* Gomont.

(25) *Lyngbya Baculum* Gomont forma.

(Fig. 24)

Essai de classification des Nostocacées Homocystées, *Journ. de Botanique*, IV, p. 354, 1890; Gomont, *Mono. Oscillariées*, II, *Ann. sci. nat.*, sér. 7, Bot., XVI, p. 123, pl. 2, fig. 17, 1892; Forti in De Toni, *Sylloge Algarum*, V, p. 259, 1907; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 1039, fig. 657, 1932.

Filaments bent and crescent-shaped, attached by the middle portion, 13.1-14.4 μ broad; sheath thin, not lamellated; trichome blue-green, 7.9-9.2 (-10.5) μ broad, not constricted at the cross-walls; cells short, up to $\frac{1}{2}$ as long as broad; apex of the trichome slightly broader, not calyptrate.

On *Enhalus*, Pamban.

This alga resembles the type in the trichome being bent and in the apex of the trichome being slightly broader, but differs from it in its mode of attachment and in the trichome being unconstricted at the cross-walls.

(26) *Lyngbya Holdenii* Forti.

(Fig. 25)

In De Toni, *Sylloge Algarum*, V, p. 260, 1907; Tilden, *Myxophyceae* in *Minnesota Algae*—I, p. 115, 1910; Geitler, *Cyanophyceae*, in Rabenhorst's *Kryptogamenflora*, XIV, p. 1040, 1932.

Filaments attached to other algae with free ends, about $7.9\ \mu$ broad; sheath up to $1\ \mu$ thick, not lamellated; trichomes ($2.6-3.9-5.3\ \mu$ broad, well constricted at the cross-walls; cells longer than or shorter than broad; apex round, not calyptrate; contents rose-red in colour.

On other algae growing on Chanks, in deep water Rameswaram.

This alga is a deep sea form and its trichomes are rose-red in colour. It agrees with the type generally but differs from the type in its trichomes being slightly broader and in the colour of the trichome being rose-red. In the latter aspect it resembles *Lyngbya Agardhii* Rabenh.

(27) *Lyngbya Nordgardhii* Wille.

(Fig. 26)

Algol. Notizen, XXVIII, *Nyt. Magazin Naturv.*, LV., p. 32, 1918; Geitler, *Cyanophyceae*, in Rabenhorst's *Kryptogamenflora*, XIV, p. 1040, 1932; Frémy, Les *Lyngbyées* de la Normandie *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 44, 1935; The Marine Algae of the Danish West Indies, *Dansk. Botanisk Arkiv.*, Bd. IX, n. 7, p. 31, 1939.

= *Lyngbya epiphytica* Wille, Algol. Notizen XXIV, *Nyt. Magazin Naturv.*, LI, p. 25, pl. 1, fig. 14-17, 1913.

= *Lyngbya Willei* Setchell and Gardner in Gardner, New Pacific Coast Marine Algae, III, *Univ. Calif. Publ., Botany*, VI, p. 468, 1918.

Filaments epiphytic; sheath thin and hyaline; trichome about $2\ \mu$ broad, slightly constricted at the cross-walls, cross-walls not granulated; cells longer than broad, up to $4\ \mu$ long; apex attenuated, round without a calyptra.

On *Lyngbya majuscula*, washed ashore, near Dhanushkodi.

Section *Eulyngbya*.

(28) *Lyngbya majuscula* Harvey.

(Fig. 27)

Phyco. Brit., pl. LXII, 1846; *Nereis Boreali-Americana*—III, p. 101, pl. XLVIIA, 1858; Kützing, *Sp. Algarum*, p. 283, 1849; *Tab. phycologicae* I, p. 49, pl. XC, fig. 1, 1849; Rabenhorst, *Fl. Eur.*

Alg., II, p. 140, 1865; Thuret, *Essai de la classification des Nostochinées*, *Ann. sci. nat.*, sér. 6, Bot., I, p. 379, 1875; Gomont, *Mono. Oscillariées*, *Ann. sci. nat.*, sér. 7, Bot., XVI, p. 131, pl. 3, fig. 3-4, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 268, 1907; Tilden, *Myxophyceae*, in *Minnesota Algae*—I, p. 123, pl. 5, fig. 42, 1910; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 1060, fig. 672c-d, 1932; Frémy, Les *Lyngbyées* de la Normandie, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 40, 1935; Boergesen and Frémy, Marine Algae from the Canary Islands, *Kgl. Dansk. Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 24, 1936; Yoneda, *Cyanophyceae of Japan*, II, *Acta Phytotax. et Geobot.*, VII, n. 2, p. 98, fig. 61, 1938; Frémy, The Marine Algae of the Danish West Indies, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 28, 1939.

= *Schizosiphon major* Kützing, *Sp. Algarum*, p. 328, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 244, 1865.

= *Lyngbya anguina* Montagne, Kützing, *Sp. Algarum*, p. 284, 1849; *Tab. phycologicae* I, p. 49, pl. 90, fig. 6, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 147, 1865.

= *Lyngbya major* Kützing, *Sp. Algarum*, p. 284, 1849; *Tab. phycologicae* I, p. 49, p. 90, fig. 8, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 140, 1865.

= *Lyngbya tropica* Kützing, *Sp. Algarum*, p. 283, 1849; *Tab. phycologicae* I, p. 49, pl. 89, fig. 6, 1849.

= *Lyngbya margaritacea* b. *homogenea* Kützing, *Sp. Algarum*, p. 283, 1849.

= *Lyngbya erosa* Liebm. in Kützing, *Sp. Algarum*, p. 284, 1849; *Tab. phycologicae* I, p. 49, pl. 90, fig. 3, 1849.

= *Lyngbya mauritanica* Montagne in Kützing *Sp. Algarum*, p. 284, 1849; *Tab. phycologicae* I, p. 49, pl. 90, fig. 4, 1849; var. *Gaudichaudiana* Montagne in Kützing, *Sp. Algarum*, p. 284, 1849.

= *Lyngbya pacifica* Kützing, *Sp. Algarum*, p. 284, 1849; *Tab. phycologicae* I, p. 49, pl. 90, fig. 7, 1849.

= *Lyngbya crispa* var. *violacea* Desm., Rabenhorst, *Fl. Eur. Alg.*, II, p. 139, 1865.

Filaments intertwining and forming a large mass, dark blue-green long, often curled variously, sometimes in a spiral manner, $25.2-52.4$ (-67.6) μ broad; sheath colourless, lamellated, up to $10\ \mu$ thick; trichome $19.3-48.3\ \mu$ broad, not constricted at the cross-walls, cross-walls not granulated; apex of the trichome rounded, not attenuated, not capitate, without calyptra; cells $3-6\ \mu$ long.

Entangled among other algae or flating in masses; Krusadai Island, Shingle Island, Pamban, Dhanuskodi, Hare Island near Tuticorin, and also from Keerimalai near Jaffna, Ceylon (leg., S. Doraiswami).

The dimensions of this alga are very varying. Krusadai Island (Fil. 32.2-48.3 (-67.6) μ br., trichome 25.7-35.9 (-14.9) μ broad), Shingle Island (Fil. up to 48.3 μ br., Trichome 28.9-35.4 μ br., sheath only 3 μ thick), Dhanushkodi (Fil. 35.4-52.4 μ br., Trichome 26.2-35.4 μ broad), Hare Island (Fil. 25.2-32.2 μ br., Trichome 19.3-25.8 μ br.).

(29) *Lyngbya confervoides* Ag.

(Fig. 28)

Gomont, Essai de classification des Nostocacées Homocystées *Journ. de Bot.*, IV, p. 354, 1890; Mono. Oscillariées, *Ann. sci. nat.*, sér. 7 Bot., XVI, p. 136-137, pl. 3, fig. 5-6, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 271, 1907; Tilden, *Myxophyceae* in *Minnesota Algae*—I, p. 119, pl. 5, fig. 39, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., *Univ. Calif. Publ.*, VIII, n. 1, p. 77, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 1061, fig. 672b, 1932; Frémy, *Les Lyngbyées de la Normandie*, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 40, 1936; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 24, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 29, 1939.

= *Leibleinia luteo-fusca* Kützing, *Phyc. gen.*, p. 221, 1842;

= *Leibleinia Cirrulus* Kützing, *Sp. Algarum*, p. 278, 1849; *Tab. phycologicae* I, p. 47, pl. 85, fig. 3, 1849.

= *Lyngbya luteo-fusca* Kützing, *Sp. Algarum*, p. 282, 1849; *Tab. phycologicae* I, p. 48, pl. 88, fig. 6, 1849; Thuret, *Essai de classification des Nostochinées*, *Ann. sci. nat.*, sér. 6, Bot., I, p. 379, 1875.

Filaments attached, 15.7-18.3 (-19.6) μ broad; sheath hyaline, homogeneous; trichome not constricted at the cross-walls, not attenuated, 13.1-14.4 μ broad; cells up to 3.9 μ long; cross-walls not granulated; apex broadly rounded, without calyptra.

Growing on *Wrangelia*, Krusadai Island.

This alga differs from the type in the cross-walls being not granulated.

(30) *Lyngbya semiplena* Ag.

(Fig. 29)

Rabenhorst, *Fl. Eur. Alg.*, II, p. 143, 1865; Thuret, *Essai de la classification des Nostochinées*, *Ann. sci. nat.*, sér. 6, Bot., I, p. 379, 1875; Holmes and Batters, *A revised list of British Marine Algae*, *Annals of Botany*, V, n. 17, p. 68, 1890; Gomont, *Essai de classification des Nostocacées Homocystées*, *Journ. de Bot.*, IV, p. 354, 1890; Mono. Oscillariées, *Ann. sci. nat.*, sér. 7, Bot., XVI, p. 139, pl. 3, fig. 7-11, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 273, 1907; Tilden, *Myxophyceae*, in *Minnesota Algae*—I, p. 118, pl. 5, fig. 38, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., *Univ. Calif. Publ.*, Botany, VIII, n. 1, p. 78, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 1061, fig. 672a, 1932; Frémy, *Les Lyngbyées de la Normandie*, *Mem. Soc. Arch. Hist. Natur. dept. Manche*, p. 41, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 29, 1939.

= *Leibleinia semiplena* Kützing, *Phyc. gen.*, p. 221, 1843; *Phyc. germ.*, p. 179, 1845; *Sp. Algarum*, p. 278, 1849; *Tab. phycologicae*, I, p. 46, p. 85, fig. 1, 1849.

= *Lyngbya tomentosa* Kützing, *Phyc. gen.*, p. 223, 1843.

= *Lyngbya Schowiana* Kützing, *Phyc. gen.*, p. 223, 1843; *Phyc. germ.*, p. 180, 1845; *Sp. Algarum*, p. 280, 1849; *Tab. phycologicae*, I, p. 47, pl. 87, fig. 2, 1849.

= *Lyngbya Schowiana* b. *tomentosa* Kützing, *Sp. Algarum*, p. 281, 1849; *Tab. phycologicae*, I, p. 47, pl. 87, fig. 3, 1849.

= *Leibleinia sordida* Kützing, *Phyc. germ.*, p. 180, 1845; *Sp. Algarum*, p. 278, 1849; *Tab. phycologicae*, I, p. 46, pl. 85, fig. 6, 1849.

= *Leibleinia caespitula* Kützing, *Sp. Algarum*, p. 278, 1849; *Tab. phycologicae*, I, p. 46, pl. 85, fig. 5, 1849.

= *Lyngbya guyanensis* Kützing, *Sp. Algarum*, p. 282, 1849; *Tab. phycologicae*, I, p. 48, p. 88, fig. 5, 1849.

= *Lyngbya lusitanica* Montagne, *Ann. sci. nat.*, sér. 4, Bot., IX, p. 149, 1858.

= *Phormidium congestum* Rabenhorst, *Fl. Eur. Alg.*, II, p. 128, 1865.

Plant mass caespitose, expanding, mucous; filaments entangled, soft, flexuous; sheaths colourless, somewhat mucous; trichomes

9.2-11.8 (-13.1) μ broad; blue-green unstricted at the cross-walls, slightly attenuated at the apices, capitate; apex with a calyptra; cells up to $\frac{1}{2}$ as long as broad; cross-walls not granulated.

On rocks, Dhanushkodi, Cape Comorin and Seven Pagodas.

Genus *Symploca* Kützing.

(31) *Symploca hydroides* Kützing.

(Fig. 30)

Sp. Algarum, p. 272, 1849; *Tab. phycologicae*, I, p. 44, pl. 76, fig. 2, 1849; Gomont, *Essai de classification des Nostocacées Homocystées*, *Journ. de Bot.*, IV, p. 354, 1890; Mono. *Oscillariées*, *Ann. sci. nat.*, sér. 7, XVI, p. 107, pl. 2, figs. 1 and 4, 1892; Forti, in De Toni, *Sylloge Algarum*, V, p. 300, 1907; Tilden, *Myxophyceae*, in *Minnesota Algae*—I, p. 129, pl. 5, fig. 49, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., *Univ. Calif. Publ. Botany*, VIII, n. 1, p. 81, pl. 1, figs. 12 and 13, 1919; Geitler, *Cyanophyceae* in Rabenhorst's *Kryptogamenflora*, XIV, p. 1119, fig. 724, 1932.

Forma minor f. nov.

Filaments forming erect dark blue-green fascicles, up to one inch high; false branching present; filaments 5.2-6.6 μ broad; sheath thin, hyaline and homogeneous; trichomes blue-green, 2.6-3.9 μ broad, usually not constricted at the cross-walls; cells shorter or longer than broad (up to twice as long as broad); apical cell rounded, not calyptrate.

On submerged rocks, Krusadai Island.

This form resembles the type in the nature and the height of fascicles and in the filaments being branched but it differs from it in having much thinner filaments and trichomes.

Genus *Microcoleus* Desmazieris.

(32) *Microcoleus chthonoplastes* Thuret.

(Fig. 31)

Essai de la classification des Nostochinées, *Ann. sci. nat.* sér. 6, *Bot.*, I, p. 378, 1875; Gomont, *Essai de classification des Nostocacées Homocystées*, *Journ. de Bot.*, IV, p. 353, 1890; Mono. *Oscilla-*

rées, *Ann. sci. nat.*, sér. 7, *Bot.*, XV, p. 353, pl. 14, fig. 5-8, 1892; Hansgirg, *Prodromus der Algenflora von Böhmen*, I, pt. 2, p. 77, 1893; Forti, in De Toni, *Sylloge Algarum*, V, p. 371, 1907; Tilden, *Myxophyceae* in *Minnesota Algae*—I, p. 155, pl. 6, fig. 28, 1910; Setchell and Gardner, *The Marine Algae of the Pacific Coast of North America*, I—Myxo., *Univ. Calif. Publ. Botany*, VIII, n. 1, p. 86, 1919; Geitler, *Cyanophyceae*, in Rabenhorst's *Kryptogamenflora*, XIV, p. 1133, fig. 739, 1932; Boergesen and Frémy, *Marine Algae from the Canary Islands*, *Kgl. Dansk. Vidensk. Selsk., Biol. Medd.*, XII, n. 5, p. 18, 1936; Frémy, *The Marine Algae of the Danish West Indies*, *Dansk Botanisk Arkiv.*, Bd. IX, n. 7, p. 56, 1939.

= *Chthonoblastus salinus* Kützing, *Phyc. gen.*, p. 197, 1843; *Phyc. germ.*, p. 165, 1845; *Sp. Algarum*, p. 262, 1849; *Tab. phycologicae* I, p. 38, pl. 58, fig. 2, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 133, 1865.

= *Microcoleus gracilis* Hassall, *British Freshwater Algae.*, p. 261, pl. 70, fig. 2, 1857; Cooke, *Brit. Freshwater Algae*, p. 255, pl. 99, fig. 1, 1882-84; Wolle, *Freshwater Algae of U.S.A.*, p. 306, pl. 203, fig. 10-11, (excluding varieties) 1887.

= *Chthonoblastus Lyngbyei* Kützing, *Phyc. gen.*, p. 197, 1843; *Phyc. germ.*, p. 165, 1845; *Sp. Algarum*, p. 262, 1849; *Tab. phycologicae*, I, p. 38, pl. 58, fig. 1, 1849;

= *Microcoleus anguiformis* Harvey in Hassall, *Brit. Freshwater Algae*, p. 261, pl. 70, fig. 1, 1845; Harvey, *Phyco. Brit.*, p. 38, 1846.

= *Chthonoblastus anguiformis* Kützing, *Sp. Algarum*, p. 262, 1849; *Tab. phycologicae*, I, p. 38, pl. 57, fig. 1, 1849; Rabenhorst, *Fl. Eur. Alg.*, II, p. 133, 1865.

= *Microcoleus salinus* c. *stratificans* (Fior. Mazz.) Rabenhorst, *Fl. Eur. Alg.*, II, p. 134, 1865.

= *Microcoleus salinus* r. *stratificans* (Fior. Mazz.) Hansgirg, *Prodromus der Algenflora von Böhmen*, II, p. 77, 1892.

Filaments intermingled with other algae, 19.3-32.2 μ broad, sometimes branched; sheath thick, in some cases annulated; trichomes blue-green nearly straight, closely packed in one or more bundles, numerous, sometimes constricted at the cross-walls, 3.5-2 μ broad; cells 3.9-7.9 μ long; apices attenuated, with a conical apical cell or sometimes not attenuated.

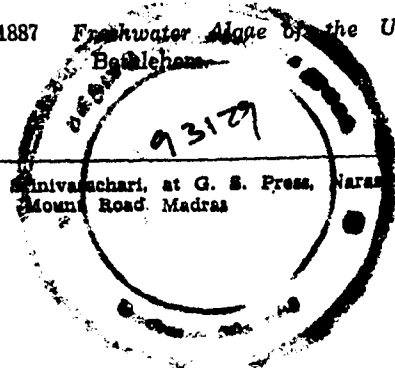
Among other algae on Chanks, Rameswaram and also from Karwar and Galle.

REFERENCES

- Batters, E.A.L. 1892 "Microchaete aeruginea sp. n." *Journal of Botany*, XXX, p. 86.
- Boergesen, F. 1935 "A list of Marine Algae from Bombay." *Kg. Danske Videnskabernes Selskabs., Biol. Medd.*, XII, 2. (reprinted).
- _____ 1936 "Some Marine Algae from Ceylon." *The Ceylon Journal of Science*, Section A, Botany, V, 12, part ii (reprinted).
- Boergesen, F., and Frémy, P. 1936 "Marine Algae from the Canary Islands, especially Tenerife and Gran Canaria, IV. Cyanophyceae," *Kg. Danske Videnskabernes Selskab., Biol. Medd.*, XII, 5. (reprinted).
- Bornet, E., and Flahault, C. 1886-1888 "Revision des Nostocacées filamenteuses Hétérocystées." *Ann. Sci., Nat.*, sér. 7, Bot., 1886—Part 1, Vol. III, pp. 323-380. 1886—Part 2, Vol. IV, pp. 343-373. 1887—Part 3, Vol. V, pp. 51-129. 1888—Part 4, Vol. VII, pp. 177-262.
- Bornet, E., and Thuret, G. 1876-1880 *Notes algologiques; recueil d'observation sur les algues*, Paris. Fasc. I, 1876. Fasc. II, 1880.
- Borzi, A. 1914, 1916, 1917 "Studi Sulle Mixoficee." *Nuovo Giorn. Bot. Ital.*, n. ser. XXIII, XXIV.
- Cleve, P.T. 1901 "Plankton from the Indian Ocean and the Malay Archipelago." *K. Sv. Vet.-Akad. Handl.*, XXXV, 5, pp. 1-58, 8 pl.
- Cooke, M.C. 1882-1884 *British Freshwater Algae*, two vols, London.
- Dixit, S.C. 1936 "The Myxophyceae of the Bombay Presidency, India—I." *Proc. Indian Acad. Sci.*, B, III, p. 93-106.
- Forti, A. 1907 *Sylloge Myxophycearum omnium hucusque cognitarum in De Toni, Sylloge Algarum*, V.
- Frémy, P. 1935 (36) "Les Lyngbyées de la Normandie." *Memoires la soc. d' Archéologie et de Histoire Naturelle de Department de la Manche*. XLVII, pp. 116-188 (reprint).
- Frémy, P. 1939 "The Marine Algae of the Danish West Indies." *Cyanophyceae Dansk Botanisk Arkiv.*, Bd. IX, n. 7. (reprint)
- Gardner, N.L. 1917 "New Pacific Coast Marine Algae I." *Univ. Calif. Publ., Botany*, VI, pp. 377-416, pl. 31-35.
- _____ 1918a "New Pacific Coast Marine Algae II." *Ibid.*, VI, pp. 429-454, pl. 36-37.
- _____ 1918b "New Pacific Coast Marine Algae III." *Ibid.*, VI, pp. 455-486, pl. 38-41.
- _____ 1919 "New Pacific Coast Marine Algae IV." *Ibid.*, VI, pp. 487-496, pl. 42.
- Geitler, L. 1932 "Cyanophyceae, in Rabenhorst's *Kryptogamenflora*, XIV, Leipzig.
- Gomont, M. 1890 "Essai de classification des Nostocacées Homocystées." *Journ. de Bot.*, IV, pp. 349-357.
- _____ 1893 "Monographie des Oscillariées." *Ann. sci. nat.*, sér. 7, Bot. XV, pp. 1-302.
- Gopala Iyer, R., and Sankara Menon, K.S. 1936 "Plankton records for the year 1929-30." *Journ. Madras Univ.*, VIII, n. 1, pp. 97-139.
- Hansgirg, A. 1892 *Prodromus der Algenflora von Böhmen*. 2, Prague.
- Harvey, W. H. 1846-51 *Phycologia Britannica*. 3 Vols., London.
- _____ 1852-58 *Nereis Boreali-Americana*. 3 Pts., Washington.
- Hassall, A.H. 1845 *A History of the British Freshwater Algae etc.*, 2 Vols., London.
- Holmes, E.M., and Batters, E.A.L. 1890 "A revised list of British Marine Algae." *Annals of Botany*, V, 67-107.
- Iyengar, M.O.P. 1927 "Krusadai Island Flora." *Bulletin of Madras Govt. Museum*, New series, Nat. Hist. Section, I, n. 1, 1-4.
- Karsten, G. 1907 "Das Indische Phytoplankton nach dem Material der Deutschen Tiefsee-Expedition 1898-99." *Wissenschaftliche Ergebnisse der Deutschen Tiefsee-Expedition auf dem Dampfer 'Valdivia', 1898-99*. Bd. II, Teil, 2, Lief. 3.
- Kirchner, O. 1900 *Schizophyceae in Engler-Prantl Natürliche Pflanzenfam.*, I, la, Leipzig.
- Kützting, F.T. 1843 *Phycologia generalis etc.*, Leipzig.
- _____ 1845 *Phycologica germanica d. i. Deutschlands Algen*. Nordhausen.

-
- 1849 *Species Algarum*, Leipzig.
-
- 1845-52 *Tabulae phycologicae*. Bd. I, II.
-
- Lemmermann, E. 1896-97 "Die Algenflora der Sandwich-Inseln"
Ergebnisse einer Reise nach dem Pacific.
H. Schauinsland, *Engl., Bot. Jahrb.* XXXIV,
607-663, pl. 7-8.
-
- 1910 *Algen I*, in *Kryptogamenflora* der Mark
Brandenburg. III, Leipzig.
-
- Martens, G.V. 1866 *Die Preussische Expedition nach Ost Asien*.
Berlin.
-
- Meneghini, J. 1841 "Monographia Nostochinearum Italicarum
addito specimine de Rivulario." *R. Acad.*
Sci., sér. 2, V, pp. 1-144.
-
- Montagne, C. 1844 "Mem. sur la coloration des Eaux de la mer
Rouge." *Ann. sci. nat.* sér. 3, Bot. II, p.
332-362.
-
- Rabenhorst, L. 1865 *Flora Europea Algarum*, II, Leipzig.
-
- Setchell, W.A. 1926 "Tahitian Algae collected by Setchell, W.A.,
C. B. Setchell and H. E. Park." *Univ.*
Calif. Publ., Botany, XII, n. 5, pp. 61-142,
pl. 7-22.
-
- Setchell, W.A., and 1919 "The Marine Algae of the Pacific Coast of
Gardner, N.L. North America, I—Myxo." *Univ. Calif.*
Publ., Botany, VIII, n. 1, pp. 1-138, pl. 1-8.
-
- Steinheil, W.A. 1834 "Cryptogames de Barberie" *Ann. Sci. nat.*,
sér. II, Bot., I.
-
- Yoneda, Y. 1938 "Cyanophyceae of Japan, II." *Acta Phytotax.*
et Geobot., VII, n. 2, pp. 88-101.
-
- Thuret, G. 1875 "Essai de la classification des Nostochinées"
Ann. Sci. nat., sér. 6, Bot., I, pp. 372-382.
-
- Tilden, J.E. 1910 *Myxophyceae in Minnesota Algae*, I,
Minnesota.
-
- Wille, N. 1913 "Algologischen Notizen XII-XXIV". *Nyt*
Magazin for Naturvidenskaberne, LI, h. 1.
-
- 1918 "Algologischen Notizen XXV-XXIX". *Ibid.*,
LVI.
-
- Wolle, F. 1887 *Freshwater Algae of the U.S.A.*, 2 vols.,
Boston.

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